RESEARCH DEPARTMENT

IMPULSIVE INTERFERENCE IN A.M. AND F.M.

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CORRIGENDUM
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G.036

1. Page 22, second line below equation 18 - delete "s" from "equations".

2. Page 22, third line below equation 18 - delete "17 and".

3. Page 22, fourth line below equation 18 - delete "s" from "equations" and delete "and 18".

4. Page 23, line 17 - delete "21" and substitute "19".

5. Page 23, equation 20 - delete right hand side and substitute -

$$\frac{\Delta f}{f_a} \cdot \sqrt{\frac{\arctan \dfrac{f_c}{f_a}}{\dfrac{f_c}{f_a} - \arctan \dfrac{f_c}{f_a}}}$$

6. Appendix 2, page 5, equation 4 - delete right hand side and substitute the expression given under correction No. 5 above.

7. Appendix 2, page 5, second and third lines below equation 4 - delete "54, 27, 59 (modified) and the remarks following equation 27", and substitute "27 and 32 for A.M. and 54 and 59 for F.M."

8. Appendix 2, page 5, Table 6 - delete the numbers in the fifth (last) column and substitute (going downwards) -

"21.6
24.2
16.6
19.2"

9. Appendix 2, page 6, last line - delete "s" from "equations" and delete "54, 27, 59 and 32 from Appendix 3" and substitute "4".

10. Appendix 8, page 2, second line below equation 7 - delete "preceding" and substitute "following".

11. Appendix 10, page 4, paragraph 4 , "The Improvement Threshold", lines 4 and 5 - delete " - in fact they are the same as the pops of interference in an A.M. system".

12. Appndix 11, page 8, second line and third column of the table - delete the entire expression and substitute the expression given under correction No. 5 above.

G.036/2

1. Page 3, equation 3 - multiply the second integral by "B".

DM/DC

RESEARCH REPORT No. G.036.
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Work carried out by - 13th July 1948.

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First draft, July 1947.

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CONTENTS

Appendices

SUMMARY

Radio interference caused by motor car ignition systems, domestic appliances and many other electrical devices is discussed. The effects produced upon it by typical A.M. and F.M. broadcast receiver circuits are dealt with both objectively and subjectively. Principal stress is laid on the aural annoyance produced by impulsive interference emerging from the loudspeakers of A.M. and F.M. sound broadcast receivers.

The theoretical considerations developed later on have been confirmed by actual measurements both metric, aural and visual. All statements made during the discussion are believed by the writers to have been adequately confirmed in practice.

For the determination of the aural annoyance of impulsive interference in A.M. reception it is found that the listener's ear may be taken as an ideal low pass filter of cut-off frequency 5 kc/s. For F.M. reception this cut-off frequency must be increased to 8 kc/s.

In view of the bandwidth limitation imposed by the ear, it is found that the audible noise coming from the receiver output, and due to

impulsive radio interference at the input, is independent of receiver bandwidth provided this is greater than that of the ear itself. This is quite general in that it applies with equal validity to both A.M. and F.M. The maximum attainable wanted modulation output from an A.M. receiver is limited by the carrier strength in that distortion will occur if the modulation amplitude exceeds that of the carrier. In a F.M. system, however, there is no practical limit to the wanted modulation output; for any deviation, however great, may be imposed upon the carrier frequency, and it is to carrier frequency deviation that the wanted modulation output is proportional. Thus the F.M. aural signal to noise ratio is proportional to carrier frequency deviation, and so is the aural noise improvement of F.M. over A.M.

This is true at all carrier levels greater than that required to exceed the FM/AM improvement threshold.

If the anti-impulsive interference characteristics of typical F.M. and A.M. systems be compared, it is found that the aural annoyance of the impulsive noise in the F.M. case is 29 db. less than in the A.M. case. This assumes that the A.M. receiver has an audio bandwidth not less than 5 kc/s, and that 50 µS pre- and de-emphasis is used in the F.M. system in conjunction with a maximum frequency deviation of $\pm$ 75 kc/s. The aural advantages to be obtained from the use of 50 µS pre- and de-emphasis are 2 db. for A.M. and 6 db. for F.M. These two figures do not allow for reduction in transmitter programme "line-up level" to avoid distortion. For a 3 db. reduction in line up level the two figures given above become - 1 db. and + 3 db. This matter has been dealt with in some detail from the subjective point of view by H.L. Kirke in the B.B.C. Quarterly of July 1946.

The A.M. and F.M. audio output peak signal to noise ratios are both independent of the radio frequency receiver bandwidth, provided that this is not less than twice the audio bandwidth. The F.M. peak audio output signal to noise ratio as distinct from the RMS or aural ratios is always 10 db. greater than in the case of A.M., assuming, of course, equal carrier strengths and audio circuits of the de-emphasis type.

In the measurement of impulse interference with a view to suppression, difficulty is experienced, particularly for low interference recurrence rates, due to the high crest factors (ratio of output pulse peak to R.M.S. value) involved. This trouble, which necessitates amplifiers of relatively high peak power handling capacity, may be mitigated by using a very narrow band receiver, thereby reducing output pulse height whilst leaving all other things the same.

The very great input interference voltages required to produce a typical and normal value of output signal to noise ratio is noted, and it is remarked that all non-linear receiving devices, such as valves, should at least in a measuring set be protected by being preceded by selective circuits. The efficacy of a selective circuit in reducing the peak value of an interfering impulsive voltage applied to it, is inversely proportional to its bandwidth.

The presence of the wanted signal carrier during reception of impulsive interference causes the amplitudes or peaks of the receiver output pulses of interference to have random values with time (that is, at each new recurrence of interference). The ear appears to be disturbed by the mean value of these random peaks rather than by the greatest of them. Since the mean value is about 4 db less than the maximum, the aural annoyance is not quite as great as might be expected from a consideration of the maximum only. This effect will not apply at very low recurrence rates as each pulse is singled out by the ear and disturbs the latter in its own right instead of as a component of a more or less even frying noise.

1. INTRODUCTION.

1.1 Definition and Description of Interference.

Electrical interference may be defined as the cause of any receiver output force other than that due to the wanted signal. It is convenient to divide it into at least the following categories:-

First, continuous wave interference. If the frequency of C.W. interference differs from that of the wanted signal by an amount less than the receiver bandwidth, it will produce a beat frequency in the output. If it is modulated, this will appear in the receiver output, and may or may not be distorted.

Secondly, random fluctuation noise, for example that produced by the Brownian motion of electrons in valves and circuits. In sound receivers this form of interference produces an even hissing noise, whilst in television receivers a "moving sand" effect is apparent on the viewing screen.

Thirdly, impulsive interference. This may be defined in "time function" terms as any electric force having a sufficiently high rate of growth or decay to shock-excite the receiver input circuits. The product of this excitation, which is itself a function of time, will progress through the receiver and emerge from the output as a time function of voltage or current, sound pressure, or light and shade, having a waveform which will be characterised by those receiver circuits having the longest response time. In spectral, or "frequency function" terms, impulsive interference may be defined as an electromotive force which has a substantially uniform spectrum over the bandwidth covered by the receiver, all of the "spectral lines" being substantially in phase at the time of occurrence of the interference. The e.m.f. at the receiver output will have a frequency spectrum characterised in amplitude and phase by the receiver circuits having the narrowest bandwidth.

The relationship between the function of time to which an electrical signal conforms and its frequency spectrum is determined uniquely by the Fourier integral (see for example Campbell and Foster, Bell System Monograph No. B.584). From the known attributes of this integral, and with the assistance of the symbolic calculus derived from it, it has been possible to develop a comparatively simple theory to explain the effects of impulsive noise upon receiver circuits, and to correlate these effects with the aural impressions of a listener.

It should be stated that virtually all cases of impulsive interference consist of repeated pulses of energy with relatively very long quiescent periods between them.

Among sources of impulsive interference may be mentioned electric railways, motor car ignition systems, and many domestic appliances such as vacuum cleaners, electric shavers, and thermostatically controlled devices such as refrigerators and some electric irons.

The subjective impression produced by impulsive interference is sometimes referred to as "annoyance," and manifests itself in sound receivers as a series of repeated clicks which with high recurrence frequencies are perceived as a frying noise. In television receivers it takes the form of specks or short bands of light on the viewing screen. The reason that the interference sounds like a frying noise rather than an even note of pulse repetition frequency (P.R.F.) is that in the presence of the wanted carrier the output pulse amplitudes have random values with time, because the time of arrival of each pulse of interference will not necessarily coincide with the same portion of the steady sinewave carrier. In other words the phase difference between the RF oscillation of the pulse and wanted carrier signal will have random values at each new pulse repetition. It may appear curious to the reader that this phenomenon should result in a frying noise rather than a steady hiss. It is considered that this is due to the high crest factor (ratio of output pulse peak to R.M.S. values) of impulsive interference. The crest factor is, of course, dependent upon receiver bandwidth and P.R.F., being high for large ratios of these two quantities.

1.2 <u>Choice of Interference Waveform for Experimental Investigation.</u>

It is reasonable to suppose that the annoyance value of impulsive interference varies with the P.R.F., the amplitude and waveshape of the receiver output. From what has been said in paragraph 1.1 it is evident that there must be an infinity of impulsive waveforms, any one of which is capable of shock-exciting the input circuits of a radio receiver. It should be emphasised that once the interference has passed through the input circuits its characteristics will be taken from these, and the exact waveform which gave rise to it is no longer of importance. The object of this report is the investigation of impulsive interference in broadcast reception, and we must therefore choose a standard input interference waveform - one which will enable us to examine all features of impulsive interference. Such a waveform must be simple and capable of representing any actual waveform by producing the same kind of receiver output as would be produced in practical cases.

The two functions of time best known to the physicist in connection with phenomena of the kind we are considering are the Heaviside unit function, or 'step' function, and its time differential, the impulsive 'p' function, known also as Dirac's function. Both these functions are valuable mathematical tools, but they are both difficult to generate in the laboratory.

The impulsive waveform chosen for the purpose of the investigations described in this report corresponds to the function of time given by the equation:

$$E(t) = 0 \text{ for } (t < 0)$$
$$E(t) = E_0 e^{-at}; \quad (t > 0)$$

This function is shown in Fig.44 as a function of time, and its frequency spectrum is shown in Fig.45.

The decrement a was chosen such that the decay of the signal over the time required for transients to build up in the circuits of a receiver was small. In other words, the experimental waveform approximates closely to a step function over a small range of time τ.

The behaviour of receiver circuits under excitation from impulsive noise sources may be considered analytically in terms of an applied signal conforming either to a unit step function or a unit impulse or 'p' function. In this report the step function type of signal has been assumed. This does not impose any restriction on the conclusions which have been reached, since the response of receiver circuits to an impulsive noise corresponding to the 'p' function can readily be deduced from its response to a step function signal; thus all signal to noise ratios involving the radio frequency, $f_0 = \frac{\omega_0}{2\pi}$, to which the receiver is tuned may be converted to unit impulse ratios by division by ω_0. The value or area of the unit impulse required to give the same noise output is from Appendix 7, equation 5.

$$B\tau = \frac{1}{\omega_0} \qquad \text{(1)}$$

where B is in amperes, volts or equivalent field strengths depending on the form of the signal.

The time duration of the impulse is assumed to be of rectangular shape. The actual interference waveform used in the experimental work is shown in Fig.2; although this appears to depart considerably from the unit step shape, its apparently rapid decay is in fact much slower than a quarter period of the receiver input circuit's natural or mid-band frequency. The slope of the leading edge is more than adequate to shock-excite these input circuits.

1.3 Prefatory Remarks on Impulsive Interference and Bandwidth.

Before going into greater detail it is instructive to discuss in general terms the effect of receiver bandwidth on impulsive noise.

For each type of radio transmission system there exists a certain suitable overall bandwidth. In amplitude modulation sound broadcasting, for example, $\pm$ 10 kc/s is usually considered adequate, whilst for 405 line television $\pm$ $2\frac{1}{2}$ Mc/s is required for double sideband reception. Frequency modulated sound broadcasting is different in that whereas the bandwidth in the receiver through which the impulsive noise must finally pass is that of the audio circuits, usually about 10 kc/s, the signal requires a wider band (which may be in the intermediate frequency circuits of a superheterodyne receiver and need not constitute the overall receiver bandwidth). Furthermore, the wider the R.F. and I.F. bands the greater may be the deviation of the wanted signal and the greater the audio output of wanted signal modulation. The audible noise spectrum, however, may be restricted to the audio bandwidth, with the corollary that the F.M. audio frequency R.M.S. output signal to noise ratio is limited only by the deviation imposed upon the wanted signal carrier, and can therefore be as great as may be desired. Unfortunately it is not possible to reap the full advantage which would accrue from very great signal deviations as other considerations such as multi-path distortion and capture effect threshold impose practical limits on signal deviation in F.M. systems.

The A.M. case is invariably such that the overall receiver bandwidth is the only one required by the mechanism of the modulation system and each individual link in the receiver chain, i.e. the R.F., I.F. and A.F. circuits may be designed to give the exact bandwidth required. There is no way, other than increasing transmitted power, in which the audio or video wanted signal can be increased without at the same time increasing the impulsive noise. The A.M. audio frequency R.M.S. output signal to noise ratio is thus limited by the ratio of strengths of wanted signal to impulsive interference.

The A.F. peak output signal to noise ratios in both A.M. and F.M. are independent of the I.F. width provided this is greater than twice the desired audio frequency bandwidth. The peak signal to noise ratio has a greater significance in A.M. than in F.M.; it is, for example, considered to be of first importance in amplitude modulated television. It will be shown later, however, that several different experiments indicate that the R.M.S. signal to noise ratio is the most important for sound broadcasting, because the aural "annoyance value" of impulsive noise seems to be dependent on energy rather than upon peak value. This has been generally accepted with regard to random fluctuation noise, but not so with impulsive noise.

1.4 Description of Test Receiver.

To clarify the effects of A.M. and F.M. receivers on impulsive interference a detailed mathematical study has been carried out on a number of representative cases. These studies sometimes followed, but were more often preceded by, laboratory experiments.

To simplify both the mathematics and the experiments the A.M./F.M. receiver used (see Fig. 1) consisted of a single stage of I.F. amplification without preceding frequency changer circuits. Two different bandwidths were obtainable by means of a switch; these were 94 kc/s and 160 kc/s. The frequency response curves are shown in Fig. 5. It is shown in Appendix 5 that the difference in transient response between a single bandpass coupled circuit and several in cascade with ideal pentode valves between them is negligible provided the overall bandwidths are the same. It follows that the conclusions reached with the aid of this single I.F. stage receiver will be valid for a multi-stage one. It will be noted that this I.F. receiver, having no frequency changer, takes no account of the effect of R.F. circuits upon the impulsive interference. This is quite legitimate if the R.F. bandwidth is wider than the I.F., for even though the R.F. circuits would distort the incoming unit steps of interference, this distortion would be very much less than the effect of the narrower I.F. circuits upon it.

When switched to the F.M. position the receiver has two resistance capacity coupled amplitude limiter stages followed by a discriminator having a bandwidth slightly greater than twice the I.F. width. The discriminator detectors and succeeding circuits have short discharge time constants, so that voltages of I.F. half bandwidth frequency may be passed on to the switchable de-emphasis circuits without distortion. The discriminator characteristics are also shown in Fig. 5.

In the A.M. switch position the I.F. bandpass coupled circuits are followed by two resistance-capacity coupled amplifiers and an "infinite impedance detector" V_7, Fig. 1a. Provision is made in the cathode circuit of V_7 to adjust the A.M. audio frequency gain to that for F.M.

2. SUBJECTIVE CONSIDERATIONS.

2.1 The Necessity for Obtaining Agreement between Objective and Subjective Experiments.

At first sight this requirement would not appear to need stressing, but so frequently painstaking laboratory experiments and theoretical work are published without final subjective confirmation that we wish to make a special point of this matter here.

We have discussed in a general way both the interference with which we propose to deal and the receiver used to investigate it, but the third and final item in the chain is vital. This is the listener. No subjective work has as yet been done with regard to television, so that we shall leave discussion of the viewer to some future occasion. Objective results on impulsive interference in A.M. will apply to television, however, so that it is probable that some careful viewing tests may be all that would be required to complete the television case.

There are very many unanswered questions regarding the human perception of sound; it is therefore of interest to enquire, at this point, whether, in listening to a broadcast programme accompanied by impulsive interference the ear is more distracted by the R.M.S. (root mean square) or the peak value of the noise.

If it is the R.M.S. value which has greater significance, this implies that it is energy rather than actual peak sound pressure which distracts the ear when listening to this particular type of noise. The following two subjective tests appear to show that energy is the distracting influence rather than peak sound pressure.

2.2 Effect of Pulse Recurrence Frequency on "Annoyance Value" of Impulsive Interference.

The experiment layout and procedure for these tests is described in Appendix 9, paragraph 3.2. The relative annoyance values of pulses of random amplitudes are plotted against pulse repetition frequency (P.R.F.) in Fig. 3. This case corresponds with normal reception conditions using either A.M. or F.M., where the amplitudes of successive pulses are random in value in the presence of the wanted carrier, as explained in paragraph 1.1. Curve A, Fig. 3, was obtained by referring the annoyance produced by each value of P.R.F. to that produced by a P.R.F. of 25 p/s. Curve B was obtained by adjusting the noise level until observers stated that the interference was "just disturbing." These two curves are averages of a small number of listeners. The "spread" of their results was so small that it appeared unnecessary to use greater numbers of observers. Comparison of Curves A and B with the straight line indicating 3 db per octave of P.R.F. shows fair agreement, and it would therefore seem that the R.M.S. value of the noise is more significant than the peak value, at any rate as regards variation of P.R.F.

The somewhat rare case of steady amplitude pulses is shown in Fig. 4, and is described in Appendix 9, paragraph 3.1. This case corresponds with A.M. or F.M. reception of impulsive interference in the absence of a wanted carrier and is, therefore, of academic rather than practical interest, though it would apply to cases of interference in audio amplifier chains. It has been stated above that the phase of the interference relative to that of the carrier is invariably random with the result that the effective amplitudes of each pulse when combined with a carrier signal have random values. If, however, no carrier is present the interference amplitudes will no longer be random and the interference output will be uniform in amplitude, that is, each output pulse will have the same amplitude. In this case, of course, the interference takes the form of a steady though distorted tone of fundamental pitch equal to the P.R.F. In Fig. 4 again there is very close agreement between the average of the listeners' observations and the 3 db per octave line. Fig. 4 was obtained by inserting programme into the audio chain.

It will be noted that both Figs. 3 and 4 do not treat with P.R.F. greater than 1000 p/s. Most interfering devices do not produce pulses with greater repetition frequencies than this. It will be observed that in Fig. 3 we do not treat with P.R.F. less than 25 p/s, because for pulses with lower repetition rates than this it was found that the annoyance value remained constant down to about 6 p/s. Thus, at very low recurrence frequencies it would appear that the ear singles out each individual pulse and is then more influenced by peak sound pressure than by energy. At values of P.R.F. less than 6 p/s the psychological state of the listener becomes important in that the degree of annoyance increases if the observer is waiting, as it were in a state of suspense, for the next click to occur.

It is interesting to note that the "spread" of observers' results becomes very great at low recurrence frequencies, but as the effect is less at low P.R.F. it is less important.

2.3 Effect of Audio Bandwidth on "Annoyance Value" of Impulsive Interference.

The practice of pre-emphasising transmitter modulation coupled with suitable receiver de-emphasis can permit of a certain reduction in receiver bandwidth whilst still maintaining a flat overall transmission frequency characteristic up to the highest audio frequencies considered necessary for satisfactory reception. This scheme provides a means of discriminating against noise without this being at the expense of upper frequency response. It is thus of interest to study the subjective effect of variation of receiver audio bandwidth upon impulsive noise.

The experimental method is described in Appendix 9, paragraph 3.4. It was found that for bandwidths between $6\frac{1}{2}$ kc/s and $1\frac{1}{2}$ kc/s the annoyance value of the noise increased at the rate of 3 db per octave of bandwidth. This result obtained in both A.M. and F.M. reception. Thus, again, it would appear that for normal audio bandwidths the ear takes more account of energy than peak sound pressure. This result would obviously not hold good for bandwidths much greater than 6 kc/s, as the ear response would be so small at such frequencies that it would be insensible to bandwidth changes. It should be remembered that the above results apply only to the case of random amplitude pulses, for change of bandwidth would produce practically no audible effect on an even approximately pure audio tone of frequency lower than the audio upper cut-off. Interference from pulses of uniform amplitude could only occur if these were inserted directly in the audio circuits and so such interference would only be encountered rarely, except wherein it occurred on long land lines.

2.4 Notes.

An interesting point brought out by the experiments on the effect of P.R.F. on what we might call the "annoyance index" was that if the listener attempted to judge annoyance without the presence of wanted programme, it was noted that although for each octave increase of P.R.F. an increase of annoyance of about 3 db was observed, yet the increment of annoyance for a sudden increase of P.R.F. of several octaves was only slightly greater than 3 db. Here we have a paradox - the whole is not equal to the sum of the parts. It was thus found that the measurement of annoyance is neither easy nor reliable without the presence of a standard of reference, distraction from which is defined as annoyance.

It should be noted that in the experiment described in paragraph 2.3 the pulse heights were not reduced to compensate for the increases brought about by the bandwidth increases. In other words, the true overall effect of bandwidth was subjectively determined. It is probably more instructive and less confusing, at least in this case, to regard an increase in bandwidth as a spectral rather than as a time function phenomenon. It is simple in this way to imagine an increase of bandwidth as a simple increase of the number of lines in the frequency spectrum accepted.

3. IMPULSIVE INTERFERENCE IN A.M.

3.1 Demodulated I.F. Output.

We now proceed with the examination of the effects produced upon a unit step receiver input by the bandpass coupled tuned I.F. circuit in the experimental receiver, Fig. 1. The demodulated output which we are about to discuss will appear across the 20 kΩ **cathode resistor of** valve V_7, Fig. 1a. If the de-emphasis switch in the anode of V_8 is in the 0 μS position, this same output will appear substantially unaltered at the output terminals of V_9, since great care has been taken to make the audio bandwidth in the 0 μS position larger than half the I.F. bandwidth, this being approximately the envelope frequency of the R.F. pulse of interference. The impulsive waveform at the input is, as has already been said, shown in Fig. 2, and will be referred to as a unit step as explained in paragraph 1.2. Before proceeding farther we must find the waveform assumed by this unit step of interference as it emerges from the I.F. circuits of the receiver. This waveform is obtained mathematically in Appendix 3, and an attempt to derive it from physical considerations follows. The transmission frequency characteristics of the bandpass coupled circuits between valves V_1 and V_2, Fig. 1a, are shown in Fig. 5. Now, the frequency mate (as defined by G.A. Campbell or Fourier transform) or frequency spectrum of a radio frequency carrier wave, amplitude modulated by a damped sinewave is almost identical with the responses shown in Fig. 5. Further, if the I.F. bandwidth is small compared with the mid-band frequency, the spectrum of the unit step input will be substantially uniform over the I.F. band; (for instance, the spectral heights of the band limits might be proportional to $\frac{1}{90 - .075}$ and $\frac{1}{90 + .075}$ for a 90 Mc/s transmission employing ± 75 kc/s deviation). Hence we may deduce that the interference noise output will consist of a carrier wave amplitude modulated by a damped sinewave (equation 8, Appendix 3) which is reproduced below -

$$e(t) = -\omega_o L e^{-\alpha t} \sin n\alpha t \; . \; \cos\omega_o t \qquad (2)$$

wherein ω_o = mid-band angular frequency,

L = inductance of each I.F. bandpass circuit coupled coils

$n = KQ$

K = coupling factor of I.F. bandpass coupled circuits

Q = quality factor of uncoupled identical I.F. circuits

$\alpha = \frac{\omega_o}{2Q} = \frac{2\pi}{F_1(n)}\Delta f$

Δf = half I.F. bandwidth for 3 db fall in response from that at f_o

$F_1(n)$ = a function of n shown in Fig. 7.

The action of detecting or demodulating this I.F. output voltage e(t) will result in a voltage $V_1(t)$ given by equation 9, Appendix 3.

$$V_1(t) = -\omega_o L e^{-\alpha t} \sin n\alpha t \qquad (3)$$

If, for instance, the circuits are at critical coupling, n = 1, and from Fig. 7 $F_1(n) = \sqrt{2}$, so the frequency of the demodulated output is $\frac{\Delta f}{\sqrt{2}}$. The function $F_1(n)$ is such that for normal values of n met with in practice the pulse output frequency is very close to $\frac{\Delta f}{\sqrt{2}}$ Fig. 6a and 6b show actual photographs of the time function $V_1(t)$ given by equation 3, whilst Fig 36 shows a graph of this expression. If we take the amplitude of the unit step interference as unity and postulate a steady wanted carrier of amplitude, η, then η is the input peak signal to noise ratio. The signal voltage amplitude at the demodulator is from equation 6 appendix 3.

$$e_1 = \eta Q \omega_o L \frac{n}{1 + n^2} = \eta \pi L \frac{n}{1 + n^2} \cdot \frac{f_o^2 F_1(n)}{\Delta f} \qquad (4)$$

If this wanted signal is amplitude modulated to a depth of 100%, this peak value of e_1 will appear at the demodulator output.

It will be observed that the signal voltage e_1 is inversely proportional to bandwidth, whilst the noise voltage V_1 is not, and so the gain of the narrower band circuits between valves V_1 and V_2, Fig. 1a, has been reduced so as to equalise the overall signal gain for the two bandwidth switch positions. The reason for this gain change is simply to arrange that when the I.F. bandwidth switch is operated the receiver programme output does not change. Thus instead of the programme output being inversely proportional to I.F. bandwidth it remains constant, and instead, the noise, which was constant, now becomes directly proportional to I.F. bandwidth. This had, of course, no effect whatever on the signal to noise ratio, in which we are interested. This gain reduction of the narrower band circuits was achieved by a proportionate reduction in the circuit inductance L. If the noise and wanted signal are received simultaneously the noise envelope given in equation 3 will have an amplitude dependent upon the exact time of arrival of the unit step interference vis à vis the phase of the signal.

As the output noise will have random amplitudes at each successive repetition of input interfering unit step it behoves us to enquire what will be the greatest value of interference, and also what will be the mean. Now the depth of modulation of the noise pulse upon the wanted signal (this depth is here assumed to be not greater than 100%) is proportional to the cosine of the phase angle between "noise pulse carrier" ($\cos\omega_o t$ in equation 2) and wanted carrier. This cosine is cos θ in Fig. 22. The maximum noise modulation is, then, given when the noise pulse carrier and wanted carrier are exactly in or out of phase. Equation 3 thus gives the maximum demodulated noise output in the presence of wanted signal. All signal to noise ratios dealt with later are calculated on a basis of this maximum value of noise. The mean of the random amplitudes has been calculated in Appendix 6. The mean of the demodulated positive noise peaks, that is, those which increase the carrier strength differs from that of the negative or "inward" peaks for low values of carrier to noise ratio, though the mean of the positive and negative means remains close to 0.63. It is this "mean of means" which is significant in the case of R.M.S. and aural signal to noise ratios. In view of these remarks it should be noted that all R.M.S. signal to noise ratios should be increased by 4 db, corresponding to 1/0.63 if we are to allow for the assumption that the ear takes account of the mean of the random values of the R.M.S. noise. Fig. 25 shows the values of the means of the outward and inward peaks of noise modulation. Figs. 6c and 6d correspond to Figs. 6a and 6b but wherein the noise and signal are both present. The various random amplitudes of successive noise output pulses arriving during the exposure time of the photograph are plainly visible; some are positive, some negative, and all intermediate values are also present. The wanted carrier to noise ratio was fairly high so that the inward and outward impulses of noise are practically equal. This corresponds with Fig. 25 for carrier to noise ratios not less than about 5. The effect of bandwidth is seen to be exactly the same as in Figs. 6a and 6b. The peak value of the demodulated I.F. output noise pulse occurs at time $t = \dfrac{\arctan n}{n\alpha}$, and if $n = 1$, for example, this time is one eighth of the period of the pulse instead of one quarter period as it would have been in the case of an undamped sinewave. This peak value is, from Appendix 3, equation 10.

$$\text{Peak } V_1 = \frac{n\omega_o L}{e^{\delta'/n}\sqrt{1+n^2}} \qquad \text{......} (5)$$

where $\delta' = \arctan n$.

If we divide 4 by 5 we obtain the demodulated I.F. outpeak peak signal to noise ratio, Appendix 3, equation 11.

$$\text{A.M. I.F. Outpeak Peak}\left(\frac{\text{Signal}}{\text{Noise}}\right) \doteq \eta\,\frac{f_o F_1(n)\; e^{\delta'/n}}{2\Delta f\sqrt{1+n^2}} \qquad \text{......} (6)$$

From this we see that the greater the bandwidth the worse is the peak signal to noise ratio. Equation 6 has been found to agree with actual measurement to within $\frac{1}{2}$ db (Appendix 2, Table 1). The R.M.S. signal to noise ratio is given by Appendix 3, equation 16.

$$\text{A.M. I.F. Output R.M.S.}\left(\frac{\text{Signal}}{\text{Noise}}\right) = \eta\,\frac{f_o}{\sqrt{f_r\,\Delta f}}\sqrt{\frac{\pi F_1(n)}{1+n^2}} \qquad \text{......} (7)$$

where f_r = P.R.F.

This ratio, as in the case of random fluctuation noise, is inversely proportional to the square root of the bandwidth. This similarity between the R.M.S. values of fluctuation and impulsive noise is due to the identity of their respective spectra over the receiver bandwidth.

The crest factor of the noise is given by Appendix 3, equation 17.

$$\text{A.M. I.F. Output Noise } \frac{(\text{Peak})}{(\text{R.M.S.})} = 2e^{-\delta'/n}\sqrt{\frac{2\pi\Delta f}{f_r F_1(n)}} \quad (8)$$

This crest factor is of importance in the measurement of impulsive noise. It has heretofore been supposed that the ear takes note of some property of the noise closely allied to either the mean value or the R.M.S., and so noise measuring receivers contain a meter which endeavours to indicate something not greatly different from the R.M.S. value. If the crest factor is high this means that the peak value of noise is much greater than that of the R.M.S. value, and so the amplifier immediately preceding the R.M.S. indicator must be capable of undistorted amplification at levels much greater than that indicated by the R.M.S. meter. Consider, for example, an A.M. sound receiver of I.F. bandwidth 10 kc/s with circuits at critical coupling and interference at a P.R.F. of 100 p/s. The crest factor for this set of conditions would be 31 db. On the other hand, a television receiver of 5 Mc/s I.F. bandwidth in similar circumstances would give rise to a crest factor of 58 db. (Both these figures are 4 db greater than the value calculated from equation 8, so as to account for the mean of random R.M.S. values). Further examples of crest factor are given in Appendix 1, Table 2.

3.2 Audio or Video Output.

We now pass on to study the effects produced upon the demodulated I.F. noise output pulse by the audio or video portion of the receiver. For simplicity this will be taken to consist of a simple de-emphasis circuit, but the arguments used will apply with almost equal validity to any normal low pass device having a substantially uniform response up to its cut-off frequency $\frac{\omega_a}{2\pi} = f_a$. An inspection of Figs. 6a, b, c and d shows that the noise pulse may be regarded as a heavily damped sinewave, in that the amplitude of the second half sinewave is very much less than that of the first. In fact, in dealing qualitatively with the effects produced upon it by the low pass device, the second half sinewave of the noise pulse may be neglected. This noise pulse enters the de-emphasis condenser and charges it to a value dependent upon the area or time integral of the pulse and the de-emphasis time constant $\frac{1}{\omega_a}$. The de-emphasis condenser then discharges at a rate determined by the time constant $\frac{1}{\omega_a}$ during the quiescent period between interfering pulses. Now the peak charge on the de-emphasis condenser is proportional to the time integral of the demodulated I.F. output pulse, and this time integral is independent of I.F. bandwidth. It therefore follows that the peak audio noise output is independent of I.F. bandwidth provided that this width is at least twice the audio bandwidth. (It is here assumed that the de-emphasis circuit does not alter signal modulation amplitude, or that if it does pre-emphasis is postulated at the transmitter to re-establish a flat overall response for signal modulation frequencies). From the foregoing statements that both noise and signal are independent of I.F. bandwidth, it may be seen that the A.F. output peak signal to noise ratio is likewise independent of I.F. bandwidth provided the latter is at least as great as double the audio width. Another way of arriving at the same conclusion is to consider, for example, a doubling of I.F. bandwidth. In this circumstance the peak I.F. output will be doubled, but since the I.F. half bandwidth is assumed to be at least as great as that of the A.F. de-emphasis circuit this peak will be halved, on account of the frequency characteristic of the de-emphasis circuit which attenuates at the rate of 6 db per octave. Here once again the peak A.F. output noise is independent of I.F. width as in the case of the signal modulation or in other words, the peak A.F. output signal to noise ratio is independent of I.F. width. Fig. 8 shows the A.F. output waveform for two values of I.F. bandwidth and three de-emphasis time constants. The time duration of the white traces in the photographs of Figs. 6, 8 and 9 is approximately 60 microseconds. It will be observed that the peak value of the waveform is practically constant even when the I.F. bandwidth changes by 1.7 to 1 and that it is practically inversely proportional to de-emphasis time constant or directly proportional to audio bandwidth. The peak A.F. output signal to noise ratio cannot be given by a single algebraic expression but may be obtained by dividing equation 4, section 3.1 by equation 21, appendix 3, and then inserting into this ratio the value of αt which satisfies the transcendental equation 22 of appendix 3. This ratio is given in appendix 2, equation 1. A quick practical way of obtaining the approximate A.F. peak output signal to noise ratio when the I.F. width is greater than twice the audio, is to multiply equation 6 by the ratio of Δf to A.F. bandwidth.

A number of experimental measurements are compared with calculations in Appendix 2, Table 2. It may be seen that the difference between calculation with this equation and measurement is of the order of 2 db in ratios around 60 db. Now the peak signal to noise ratio is useful to know from the point of view of noise measurement and crest factor determination but as stated in section 2.1 it is not the signal to noise ratio heard by the ear. The aural signal to noise ratio may be obtained from the R.M.S. signal to noise ratio by taking into account the response shape of the ear, although this in its turn is subject to a great number of variables of which sound loudness is one of the most important. The audio frequency R.M.S. signal to noise ratio is a measure of the A.F. output noise energy, and since the energy spectrum of the input noise or impulsive interference is substantially flat over the receiver bandwidth the A.F. output energy will be proportional to the overall (therefore narrowest) bandwidth in the receiver. If the I.F. bandwidth is not less than twice that of the A.F. circuits, then the A.F. output R.M.S. signal to noise ratio will be independent of I.F. bandwidth and inversely proportional to the square root of the audio bandwidth, equation 27, appendix 3 reproduced below:

$$\text{A.M. A.F. Output R.M.S.} \frac{(\text{Signal})}{(\text{Noise})} = \eta \frac{\pi}{2} \cdot \frac{f_o}{\sqrt{f_r \cdot f_a}} \quad \text{.........} (9)$$

A general formula relating to the relative values of I.F. and A.F. bandwidths from which equation 9, above, was obtained, is given in Appendix 3 as equation 26. Equation 9, and the equation from which it is derived, assume that the audio portion of the receiver consists of an ideal low pass filter - not a de-emphasis circuit. The cut-off frequency of this filter is taken as f_a. If, in fact, de-emphasis of time constant $\frac{1}{2\pi f_a}$ be used instead, it is necessary to multiply equation 9 by $\sqrt{\frac{2}{\pi}}$ because the energy bandwidth of a de-emphasis circuit of time constant $\frac{1}{2\pi f_a}$ corresponds with that of a low pass filter of cut-off frequency $\frac{\pi}{2} f_a$.

Some comparisons between calculation and measurement are shown in Appendix 2, Table 3. De-emphasis was actually used, and instead of multiplying equation 9 by $\sqrt{\frac{2}{\pi}}$ to account for the larger energy bandwidth of de-emphasis the value of f_a put into this equation was itself multiplied by $\frac{\pi}{2}$ and referred to in the table as "effective f_a." The discrepancy between theory and practice is thus seen to be not greater than $2\frac{1}{2}$ db. at ratios corresponding to some 70 db.

Now the actual aural signal to noise ratio can be calculated from equation 9 in which f_a has a value determined from a consideration of the energy bandwidth of a de-emphasis circuit in cascade with a low pass filter of energy width approximating to 5 kc/s, the latter resulting from a determination of the energy width of the C.C.I.F. ear response curve, Appendix 3, equation 32. In these circumstances the aural signal to noise ratio is given by equation 10 below which is a combination of equation 9 above and equation 32, Appendix 3.

$$\text{A.M. Aural} \frac{(\text{Signal})}{(\text{Noise})} = \eta \frac{\pi}{2} \frac{f_o}{\sqrt{f_r f_a \arctan \frac{5}{f_a}}} \quad \text{.........} (10)$$

wherein all frequencies are in kilocycles per second.

An attempt to check equation 10 by listening tests was a failure, in that observers were not able consistently to judge equality of loudness between tone or programme on the one hand and the frying noise produced by pulses of random amplitudes on the other. This was an attempt to judge, aurally, an absolute signal to noise ratio. Listeners gave answers both higher and lower than the calculated values but with so great a statistical spread as to be useless.

In Fig.12 a calculated aural signal to noise ratio is shown. The audio portion of the receiver is taken to be an ideal low pass filter of cut-off 5 kc/s which, according to the C.C.I.F. ear curve, represents the energy bandwidth of the listener's ear when listening to an A.M. receiver having an audio width not less than that of the ear itself. The carrier to noise ratio, η, is arbitrarily taken as unity. It may be seen that the simplified equation 9 holds with reasonable accuracy down to I.F. bandwidths not less than double the audio width. Exactly the same curve as Fig.12 would result from the application of an impulse of 0.038 microcoulombs instead of the unit step actually chosen.

Regarding the input interference as of unit step form for the moment, and assuming that a reasonable output signal to noise ratio might be of the order of 40 db, we see from Fig.12 that this would be obtained from an input noise to signal ratio of 73 db (the output signal to noise ratio for $\eta = 1$) minus 40 db. = 33 db. Thus if a 40 db. output signal to noise ratio were obtained with an input signal of, for example, 50 millivolts then the input noise would be 33 db. greater than 50 millivolts or $2\frac{1}{4}$ volts. The noise reducing properties of sound receivers are thus seen to be considerable and it is worth noting that in cases of severe interference of an impulsive nature this may amount to field strengths of many peak volts per metre. This applies to F.M. with even greater force, as shown in Fig.13.

Some cases of audio frequency R.M.S. signal to noise ratios of general interest are shown in Appendix 1, Table 1.

3.3 Effect of Mistuning.

All the foregoing assumes that the receiver is accurately tuned to the incoming wanted signal carrier frequency. The effect of mistuning is shown in Fig.10a. There is, in this case, a frequency difference component between the wanted carrier and the $\cos\omega_0 t$ term in equation 2. because the mis-tuning will result in the wanted carrier frequency being off centre whilst the noise will always remain at the frequency $\frac{\omega_0}{2\pi}$. The beat note between carrier and the radio frequency oscillation of the noise pulse will not have zero frequency so that the envelope of the noise radio frequency oscillation will not be simply that given by equation 3 but will be modulated by the frequency difference between carrier and noise pulse R.F. oscillation, that is, by the amount of the mis-tune. An extreme mistune results in a slight reduction of noise due to a partial conversion of a unidirectional pulse into positive and negative pulses, but this process will be described later when dealing with F.M. as it is of first importance in that case.

4. IMPULSIVE INTERFERENCE IN F.M.

4.1 Demodulated I.F. Output.

The frequency modulation receiver used for the tests is shown in Fig.1. The valves V_2 and V_3 Fig.1a are resistance-capacity coupled amplifiers. V_4 is a limiter and V_5 is the limiter-discriminator feed valve. The demodulated frequency modulated I.F. output is fed from the discriminator detector V_6 on to the grid of the A.F. amplifier V_8 with de-emphasis circuits in its anode as described elsewhere. Fig. 5 shows the discriminator characteristics.

In what follows we shall assume that the maximum phase angle displacement of the resultant vector of carrier and impulsive interference is small enough for the sine, the tangent and the angle to be taken as equal, Fig. 16. This restriction is not as severe as might be supposed, since input noise to signal ratios of up to 46 db. still maintain a linear relationship between input and output signal to noise ratios as would be predicted by subsequent theory, at any rate in a typical case, Fig. 11.

Now in the case of a F.M. receiver the I.F. noise voltage given by equation 2 will add vectorially with the carrier voltage, forming a resultant which will vary in both phase and amplitude. The amplitude variations will be eliminated by the limiters V_4 and V_5, Fig. 1a. The phase variations of this resultant will give rise to a noise frequency modulation proportional to the rate of change of this phase angle.

We shall assume, as we did in the A.M. case, that the receiver is correctly tuned to the wanted F.M. signal (assumed unmodulated for simplicity). A case of mis-tuning is shown in Fig. 10b. As in the A.M. case, slight mistuning has a negligible effect on the noise output though it may, of course, have a considerable effect on the wanted signal.

By virtue of the assumption of correct tuning, the noise oscillation radio carrier frequency $\frac{\omega_o}{2\pi}$ equation 2, is equal to that of the wanted signal. Thus the resultant of wanted carrier and noise is the composition, not of a vector rotating around another, but of a vector having for each repeated pulse of interference a constant phase angle, γ, with the carrier component, and this carrier component, Fig. 16. The noise vector will thus, as it were, shoot out from the tip of the carrier vector at any angle, γ, chosen at random. The law of increase of amplitude of the noise vector is given by equation 3. Since the maximum phase displacement of the resultant vector is assumed to be small, the angle between noise and carrier vectors which will give rise to maximum rate of change of phase is $\frac{\pi}{2}$ minus phase displacement, Appendix 3 and Fig. 16.

If A is the discriminator co-efficient in volts per cycle per second, it follows that demodulated noise output from the discriminator will be proportional to the product of A and the rate of change of phase displacement of the resultant of carrier and noise vectors. From Appendix 3; equation 37, this demodulated noise output from the discriminator is

$$V_2(t) = \frac{A\alpha(1+n^2)^{3/2}}{2\pi\eta nQ} e^{-\alpha t} \sin(n\alpha t-\delta') \quad \text{.......................(11)}$$

Figs. 6e and 6f show this I.F. output voltage. As in the A.M. case the various random amplitude pulses occurring during the photograph exposure time may be plainly seen. These random amplitudes are due to the random phase angles between noise and carrier vectors at each recurrence of interference.

If we refer, for a moment, to Fig. 6a for example, and follow the variation of the slope of this waveform we see that from an initial slope of zero it falls to a minimum ("negative maximum"), then to zero when the waveform negative peak is attained, and rises once more to a positive maximum during the upward or trailing edge of the pulse and finally down to zero again. This is effectively a description of the frequency modulated I.F. output waveform shown in Fig. 6e. If, of all the random pulses shown in this picture, we consider one of the two outer or maximum ones, and choose that one which starts to go negative or downwards in the photograph, we see that it reaches a "negative maximum" at first, then rises to zero and then passes through a positive maximum before re-attaining zero again.

Strictly speaking, equation 11 assumes an aperiodic discriminator, that is a device having a sloping rectilinear characteristic in the amplitude frequency plane without any selectivity or limit. If, as in normal practice, the discriminator has a bandwidth of the order of twice that of the I.F. circuits, equation 11 gives a reasonable approximation to the results obtained. Comparison of Fig. 6e with 6f shows that the peak discriminator output is approaching proportionality to the square of the I.F. width. This is simply because the F.M. output is proportional to the product of the slope and height of the A.M. pulse, and both these quantities vary directly with bandwidth as described in section 3.1.

The peak or maximum value of equation 11 occurs at time $t = 0$ and is given by Appendix 3, equation 38.

$$\text{Peak } V_2 = \frac{A\alpha(1 + n^2)}{2\pi^2 \eta f_o} \qquad (12)$$

The signal modulation output from an ideal discriminator is proportional to the deviation of its carrier frequency. We shall assume in all that follows that 100% or full modulation involves a complete utilisation of the I.F. half bandwidth to the limiting frequencies at which the response has decreased from its mid-band value by 3 db. This frequency swing is $|\Delta f|$ (see Fig. 7).

Thus the modulation output from the discriminator for 100% modulation is

$$e_2 = A\,\Delta f \qquad (13)$$

Dividing 13 by 12 gives

$$\text{F.M. I.F. Output Peak} \frac{(\text{Signal})}{(\text{Noise})} = \eta \frac{f_o F_1^{\,2}(n)}{2\,(1 + n^2)\,\Delta f} \quad (14)$$

Comparing equation 14 with 7 we see that although the F.M. peak noise is proportional to the square of the bandwidth, since the modulation is proportional to bandwidth, the peak signal to noise ratio is similar to the A.M. case in being inversely proportional to it. Equation 14 has been found to agree with actual measurements to within 3 db.,(Appendix 2, Table 1).

The I.F. output R.M.S. signal to noise ratio is given by equation 44, Appendix 3.

$$\text{F.M. I.F. Output R.M.S.} \frac{(\text{Signal})}{(\text{Noise})} = \eta \frac{f_o}{1 + n^2} \sqrt{\frac{\pi F_1^{\,3}(n)}{f_r \cdot \Delta f}} \quad (15)$$

This ratio is similar to the A.M. case with regard to I.F. bandwidth. This means that the advantage of wide band F.M. over A.M. is not obtained until the interfering noise meets the filtration of the narrower band audio circuits. We shall deal with this later. The frequency modulated I.F. output noise crest factor is from Appendix 3, equation 45

$$\text{F.M. I.F. Output Noise} \frac{(\text{Peak})}{(\text{R.M.S.})} = 2\sqrt{\frac{2\pi\ \Delta f}{f_r \cdot F_1(n)}} \quad (16)$$

This F.M. crest factor exceeds its A.M. counterpart by the multiplying factor $e^{\delta/n}$, which for critically coupled circuits ($n = 1$) becomes 2.2. Thus the measurement of frequency modulated R.M.S. noise requires more than twice the range of linear amplification than that required for A.M.

4.2 <u>Audio Output.</u>

It should be noted that Figs. 6e and 6f differ from Figs. 6c and 6d in two fundamental respects. First, as already stated, the I.F. output noise pulse height is proportional to the square of the I.F. bandwidth. Secondly, whereas in A.M. (Figs. 6c and 6d) there is virtually one single pulse for each successive repetition of impulsive interference, in F.M. (Figs. 6e and 6f) there are two pulses of opposite polarity. Furthermore, the time integrals of these two F.M. pulses are nearly equal so that though the first pulse charges the de-emphasis condenser in a manner similar to that described for A.M. in section 3.2 the second pulse discharges this condenser. The smaller charge still left after this process decreases exponentially in the normal manner. For a theoretical I.F. to A.F. half bandwidth ratio of unity, this charge and discharge process gives F.M. an advantage over A.M. of 5 db. in A.F. output R.M.S. signal to noise ratio.

The frequency modulated A.F. output peak signal to noise ratio is, like the A.M. case, independent of I.F. bandwidth, because although the F.M. A.F. output noise peak is proportional to I.F. width, so is the demodulated wanted signal. This is because we have assumed that the wanted signal deviation is made equal to the I.F. half bandwidth, Δ f. The reason that the frequency modulated A.F. output noise peak is proportional to I.F. bandwidth is simply this:- on emerging from the discriminator it is proportional to the square of the I.F. bandwidth; the attenuation of the de-emphasis circuit to frequencies beyond its pass range is 6 db. per octave, and in consequence this square law is reduced to a linear law as in the case of A.M. in section 3.2.

Fig. 9 shows the A.F. output waveform for various I.F. bandwidths and de-emphasis time constants. It should be remembered that the durations of the pulses in Fig. 9 are such that they are per se practically inaudible. It is only the slow decay voltages which are heard. As in the A.M. case, it is not possible to give a single algebraic expression for the frequency modulated A.F. output peak signal to noise ratio.

If the value of at which satisfies the transcendental equation 49, appendix 3 be inserted into equation 2 appendix 2 the desired ratio may be obtained. The equation thus developed agrees with actual measurements to within about 2 db. as shown in Appendix 2, Table 2. It is perhaps useful to remember that since both A.M. and F.M. audio frequency output peak signal to noise ratios are independent of I.F. bandwidth (provided this is at least double the audio width), whatever the maximum deviation employed in F.M. this ratio is always of the order of 10 db. better than A.M.

Once again, it is the R.M.S. signal to noise ratio which has greater significance from the aural point of view. This will now be discussed. Clearly for the same reason as explained in the A.M. case the F.M. audio frequency output R.M.S. noise will be independent of I.F. bandwidth and dependent upon that of the A.F. circuits.

The frequency modulated A.F. output R.M.S. signal to noise ratio is given by equation 54, Appendix 3.

$$\text{F.M. A.F. Output R.M.S.} \left(\frac{\text{Signal}}{\text{Noise}}\right) = \eta \, \frac{\pi\sqrt{3}}{2} \cdot \frac{f_o \, \Delta f}{\sqrt{f_r \cdot f_a^{3}}} \quad \ldots\ldots\ldots\ldots (17)$$

This equation is the limiting value of equation 53, appendix 3 when the I.F. width is greater than twice the A.F. width. Fig. 13 shows that the approximate equation 17 is adequate for all normal applications. The fact that the audio bandwidth enters this formula with a three halves power is due to the triangular spectrum of F.M. noise. This results, of course, from the differentiation of the phase modulation of the resultant of carrier and noise vectors, appendix 3 equations 36 and 37. The R.M.S. value of such a triangular spectrum will involve the square root of the integral of the squared spectrum and this will result in the square root of the cube of the band limit. As in the A.M. case, equation 17 assumes an ideal audio low pass filter rather than a de-emphasis circuit. If de-emphasis is in fact used, the R.M.S. signal to noise ratio would result from an effective audio bandwidth equal to the I.F. half bandwidth. This state of affairs results from the combination of two triangular shaped responses, Fig.17. The F.M. audio noise increases linearly with frequency, whilst above its cut-off the de-emphasis response has a linear decrease. The combination of the two results in a uniform spectrum which would extend indefinitely were it not for the I.F. circuits which in their selectivity form the ultimate limit to this noise spectrum. It is thus necessary to include an ideal low pass filter in cascade with the de-emphasis in order to limit the overall noise spectrum. Now this is just what the listener's ear does. For triangular noise the effective energy bandwidth of the ear is about 8 kc/s. From Figs. 17 and 18 and appendix 3, equation 59 we may re-write equation 17 for the aural case as

$$\text{F.M. Aural}\ \frac{(\text{Signal})}{(\text{Noise})} = \eta\,\frac{\pi}{2}\cdot\frac{f_o\,\Delta f}{\sqrt{f_r\cdot f_a^{3}\left(\frac{8}{f_a}-\arctan\frac{8}{f_a}\right)}} \qquad \text{..........(18)}$$

wherein all frequencies are in kilocycles per second. Again, as with A.M. it was not found possible to make a sufficiently precise aural check of equations 17 and 18 worthy of recording. Fig. 13 is an example of application of equation 18. The agreement between equations 17 and 18 on the one hand and actual measurements on the other is shown to be within 4 db. in Appendix 2, Table 4.

Table 1, Appendix 1 shows two frequency modulated R.M.S. aural output signal to noise ratios of general interest.

5. COMPARISON BETWEEN F.M. AND A.M.

5.1 R.M.S. A.F. Noise Output Comparison. Low Pass Filter Audio Circuits.

We shall first compare the R.M.S. A.F. output signal to impulsive noise ratios of a F.M. and an A.M. receiver each having identical A.F. bandwidths. We assume that neither I.F. bandwidth is less than twice the common A.F. bandwidth which in its turn is limited by ideal low pass filters of cut-off f_a. Appendix 2 equation 3 shows that the ratio of signal to noise ratios is

$$\text{R.M.S. Signal/Noise}\ \frac{(\text{F.M.})}{(\text{A.M.})} = \sqrt{3}\cdot\frac{\Delta f}{f_a} \qquad \text{..........(19)}$$

where Δf applies to the F.M. receiver only.

This expression agrees with measurements to within $2\frac{1}{2}$ db. (Appendix 2, table 5). It is interesting to note that the above result is identical with the ratio of the R.M.S. signal to random fluctuation noise ratios which has been calculated by M.G. Crosby in Proc. I.R.E. April 1937. This is obvious if it be remembered that the spectra of the two varieties of noise are the same over the relatively narrow receiver overall bandwidth as was stated in section 3.1. Another way of obtaining Crosby's result is to assume, first that the I.F. width is much greater than the A.F. In this case the A.M. noise pulse entering the A.F. circuits may be regarded as a Heaviside unit impulse having a uniform spectrum or "frequency mate." This obviously results in an audio noise spectrum which is uniform up to the audio cut-off. Now the F.M. noise pulse entering the A.F. circuits is the time derivative of the A.M. pulse and is therefore a double pulse, each half having opposite polarity. The "frequency mate" or spectrum of such a pulse is obtained from that of the A.M. pulse by a multiplication by a Heaviside operator $p = j\omega$. Thus the F.M. pulse spectrum is triangular and is again limited by the audio cut-off. So for a theoretical ratio of Δf to f_a of unity, the energy ratio of the two spectra may be shown to be 3 and the R.M.S. ratio to be $\sqrt{3}$. If Δf exceeds f_a then whereas neither the F.M. nor A.M. noises alter, the F.M. signal increases in proportion, thus we obtain equation 21 by a different method of reasoning.

Equation 19 will not represent the aural improvement of F.M. over A.M. because no account has been taken of the characteristics of the listener's ear.

5.2. <u>R.M.S. A.F. Noise Output Comparison. Audio Circuits constituted by De-emphasis followed by Low Pass Filters.</u>

If we assume that the two receivers have identical audio circuits as in section 5.1 but that this time they are constituted by de-emphasis in cascade with a low pass filter of cut-off f_c, then equation 19 becomes (Appendix 2 equation 4)

$$\text{R.M.S. Signal/Noise} \left(\frac{\text{F.M.}}{\text{A.M.}}\right) = \frac{\Delta f}{f_a} \cdot \frac{\frac{\pi}{2}}{\sqrt{\frac{f_c}{f_a} - \arctan\frac{f_c}{f_a}}} \qquad \text{.........(20)}$$

$$\text{wherein } f_a = \frac{1}{2\pi \times \text{de-emphasis time constant}}$$

Laboratory tests show that agreement between equation 20 and actual measurement is within 3 db. (Appendix 2, Table 6).

5.3 <u>Aural R.M.S. A.F. Noise Output Comparison. Audio Circuits constituted by Identical De-Emphasis.</u>

If f_a is related to the de-emphasis time constant by the relationship given below equation 20 we obtain

$$\text{Aural Signal/Noise} \left(\frac{\text{F.M.}}{\text{A.M.}}\right) = \frac{\Delta f}{f_a} \cdot \sqrt{\frac{\arctan\frac{5}{f_a}}{\frac{8}{f_a} - \arctan\frac{8}{f_a}}} \qquad \text{.........(21)}$$

wherein all frequencies must be in kilocycles per second. This equation is obtained from Appendix 2, equation 5. Equation 21 is found to agree with aural measurements to within 4 db at the worst, but frequently to within as close as 1 db.

5.4 <u>Aural R.M.S. Noise Output Comparison. A.M. Audio Circuits having wider bandwidth than the Ear. F.M. Audio Circuits constituted only by de-emphasis.</u>

From Appendix 2, equation 6

$$\text{Aural Signal/Noise} \left(\frac{\text{F.M.}}{\text{A.M.}}\right) = \frac{\sqrt{5}\ \Delta f}{\sqrt{f_a^{\,3}\left(\frac{8}{f_a} - \arctan\frac{8}{f_a}\right)}} \qquad (22)$$

with all frequencies in kilocycles per second, and f_a being defined in equation 20. Appendix 2, table 8 shows that expression 22 does not differ from aural measurements by more than $1\frac{1}{2}$ db. For a 50 microsecond de-emphasis, f_a becomes 3.18 kc/s, and for a deviation of ±75 kc/s, a typical F.M./A.M. aural improvement is 28 db. Fig. 14 is a plot of expression 22 showing the variation of F.M./A.M. aural improvement with I.F. bandwidth, $2\ \Delta f$. This figure is also the quotient of Figs. 13 and 12.

5.5 <u>F.M./A.M. Improvement Threshold.</u>

This is discussed in Appendix 10. Impulsive interference in F.M. sounds like a succession of clicks, if the P.R.F. is sufficiently low and the signal strength sufficiently high. If the signal strength be progressively diminished the clicks will give way to a succession of pops having a pronounced bass component as distinct from the clicks. ~~The proportion of pops to clicks will increase until the improvement of F.M. reception over A.M. will dwindle to zero. The pops, with their much increased bass content are much more annoying than clicks, so that the apparent reduction of signal to noise ratio is much greater than the actual percentage of pops to pops and clicks.~~

Pops have a uniform audio noise spectrum as distinct from the triangular spectrum associated with clicks and such a uniform spectrum can only be obtained in F.M. reception when the vector resultant of carrier and noise makes a complete rotation of $2\,m\,\pi$ radians, where m is integer. This can only happen when

(a) the A.M. noise pulse peak exceeds the carrier peak at limiter input, and

(b) the noise pulse vector is rotating with respect to the carrier vector.

Condition (b) can occur either during frequency modulation; or, in the absence or presence of modulation, in a receiver having bandpass coupled circuits in its I.F. stage or stages. A severe mis-tune of wanted carrier

CORRIGENDUM TO G.036. PAGE 24, PARAGRAPH 5.5.

The following shall replace the fourth and fifth sentences of that paragraph.

The proportion of pops to pops and clicks will increase up to 50%. The improvement of FM reception over AM decreases rapidly at first. If the signal strength be further diminished the FM noise level will increase almost imperceptibly whilst the AM noise level will increase linearly. Thus the FM/AM improvement which had diminished to a fairly small figure, will now increase once again and according to a simplified theory, can increase without limit. The pops with their much increased base content are much more annoying than clicks, so that the apparent reduction of signal to noise ratio is much greater than the actual percentage of pops to pops and clicks.

signal can similarly result in pops.

From section 3.1 equation 6, we see that the R.F. input peak carrier to noise ratio required to obtain the F.M./A.M. improvement threshold is

$$\eta = \frac{\sqrt{1 + n^2}}{Qe^{\delta'/n}} \qquad (6)$$

For $n = \sqrt{2}$ and $Q = 50$, the case quoted in Appendix 10, we find $\eta = .0177$ which corresponds to a R.F. input peak noise to signal ratio of 35 db.

6. ASSESSMENT OF ANNOYANCE OF IMPULSIVE INTERFERENCE IN THE V.H.F. BAND.

6.1 Services to be protected.

It would appear, at the present time, that the V.H.F. frequency band, covering the range 30 to 300 Mc/s, will be used for radio location and other civil airline radio services, narrow band A.M. for television sound, very wide band A.M. for television and wide band F.M. for television sound and for sound broadcasting.

It should be possible from information contained in this report to deduce what kind of measurements of impulsive interference would be suitable for protection of the above radio services, but particularly in connection with the broadcasting services in which we are principally interested.

6.2 Ideal Noise Measuring Set.

As has already been stated no subjective visual experiments have been undertaken with regard to the effect of impulsive interference on television. It may safely be said, however, that experience shows that if an interfering device be fitted with suppressors which reduce its annoyance on A.M. sound reception to an acceptable value, it will probably also be acceptable for vision.

Since the improvement of wide band F.M. over A.M. in signal to noise ratios is known, it becomes evident that interference may in general be measured and judged on ordinary A.M. sound reception principles. Thus an A.M. measuring set designed from the point of view of indicating directly or indirectly by means of appropriate calibration and computation, the subjective aural results of a given interference would also be suitable for television and wide band F.M.

The ideal noise measuring set would thus have an I.F. bandwidth of 10 kc/s and an A.F. width of 5 kc/s followed by a R.M.S. meter of, for example, thermocouple type. An input attenuator would ensure that the I.F. and subsequent circuits were always loaded by the same signal amplitude, this being determined by a peak meter at the I.F. output or demodulator input terminals. Such a receiver, without automatic gain control, would be satisfactory for values of P.R.F. down to about 25 p/s. For values of P.R.F. a little below this figure the annoyance should be taken as that for a P.R.F. of 25 p/s, thus below this value it would be necessary to measure P.R.F. and to account for it by deleting its effect from the R.M.S. meter reading which would otherwise give an incorrect result as far as annoyance is concerned. Since the peak to mean ratio of random amplitude pulses is known (Appendix 6), and in view of the fact that it has been shown by listening tests that annoyance is a function of the mean value of these random pulses it does not appear to be necessary to work with an artificially inserted wanted carrier, as is sometimes done in order to randomise the pulses, as well as with the interference to be measured. All that is necessary is to subtract 4 db. from the R.M.S. indicator reading, because in the absence of carrier, the interfering pulse output will be a maximum all the time instead of having random amplitudes; and so the R.M.S. indicator will read 4 db. more interference than the mean value which represents the aural effect. These remarks might not apply when protecting a television service from interference.

This report does not deal with values of P.R.F. below about 6 p/s as no subjective tests have been made at such low repetition rates. As the spectra of the more usual types of impulsive interference do not change rapidly with signal frequency it would not appear necessary to alter the receiver bandwidth for use in the different wavelength bands as a certain amount of local oscillator drift would have a negligible effect on impulsive interference output. Thus as already stated a V.H.F. measuring set could retain the same bandwidth as called for by existing international specifications which are limited, at present, to long and medium waves.

6.3 A Practical Noise Measuring Set.

As the effect of receiver bandwidth is capable of detailed quantitative prediction, it is suggested that a practical noise measuring receiver for V.H.F. use should comprise R.F., I.F. and A.F. circuits with progressively diminishing bandwidths. The A.F. width should be as small as possible to obviate as far as is feasible difficulties due to high crest factor. Thus an A.F. width of 1 kc/s would not be too small. A R.F. attenuator should precede the first valve but should itself be preceded by an input selective circuit to reduce the input unit step or impulse of interference to a manageable value. All valve grids should be protected from unnecessarily high crest factors by selective circuits. In particular, no unselective (for example, resistive) attenuators should be put between frequency changer and I.F. valves.

The I.F. output from this receiver should contain a peak detector and a normal type of demodulator with a discharge time constant of the order of $\frac{1}{2\pi\ \Delta f}$. The peak detector would then be used as a "red line" reading device to enable the input unit step or impulse amplitudes to be determined from the input attenuator setting. After this adjustment the pulse amplitude passing through to the demodulator would be kept constant. The R.M.S. indicator following the narrow audio circuits could then be a measure of P.R.F. The input unit step and its P.R.F. would then be used in conjunction with the appropriate formulae to determine what annoyance would be caused by the measured interference upon a wanted signal of known amplitude. An A.F. attenuator immediately preceding the R.M.S. meter would permit of "red line" adjustment of the latter for a standard annoyance value obtained for pre-determined settings of R.F. and A.F. attenuators.

H L Kirke

(H. L. Kirke)

Appendix 1.

Some Typical Examples of Signal to Noise Ratios Calculated from the Formulae derived in Appendix 3.

1. Introductory Remarks.

Signal to noise ratios for various typical receivers have been calculated from the formulae derived in Appendix 3. These ratios are calculated and discussed in this appendix.

Wherever possible both unit step and unit impulse forms of input interference are postulated. In practice, both types may be encountered as well as other kinds giving rise to exceedingly complicated spectra. It is invariably true, however, that the variation of shape of the spectra with frequency is slow compared with typical receiver bandwidths. This applies even with wide band receivers for television. This fact makes it possible to standardise all impulsive interference in terms of one or two fundamental waveforms as described elsewhere.

2. Some Typical Signal to Noise Ratio Comparisons.

Table 1 gives the audio frequency output root mean square signal to noise ratios for various practical receiver designs for a one to one input-signal-amplitude to input-unit-step ratio. Included also are the signal to noise ratios based on that value of unit impulse which produces the same A.F. output R.M.S. signal to noise ratio as the unit step case for a receiver tuned to the radio frequency of 1 Mc/s. This is quite an arbitrary choice but as good as any other. The unit impulse ratios show explicitly the effect of receiver bandwidth as they are independent of the radio frequency, f_o, to which the receiver may be tuned. The value of this unit impulse is, from appendix 7, equation 5, $\frac{1}{2\pi f_o} = 0.16 \times 10^{-6}$ ampere-seconds or coulombs wherein $f_o = 1$ Mc/s.

Table 1 is calculated from the following equations:

Lines 1 to 7 from equation 26, appendix 3
Line 8 from equations 6, 21 and 22, appendix 3
Lines 9 to 12 from equation 16, appendix 3
Lines 13 and 14 from equation 27, appendix 3
Lines 15 and 16 from equations 54 and 59, appendix 3.

The actual R.M.S. A.F. output signal to noise ratios are increased by 4 db. (see appendix 6, section 2) to allow for the peak to mean ratio of random amplitude pulses and are then decreased by 8 db. to allow for the fact that in sound broadcasting the average programme level is 8 db. below 100% modulation. The television cases, however, are as calculated by the formulae quoted and are not increased by 4 db. nor decreased by 8 db.

Table 1.

Column No.	1	2	3	4	5	6	7	8	9
Line No.	Radio Frequency Received	Type of Mod.	I.F. $\frac{1}{2}$ Band Δf	Coupling Parameter n of IF ccts	Audio Band f_a	P.R.F. f_r	Unit Step Effective RMS Signal/ Noise	Unit Impulse Effective RMS Signal/ Noise	Remarks
1.	1 Mc/s	A.M.	± 5 kc/s	$\sqrt{2}$	10 kc/s	1000 p/s	52.7 db	52.7 db	aural case
2.	"	"	"	"	"	25	68.7	68.7	" "
3.	4.2	"	"	"	"	1000	65.1	52.7	" "
4.	"	"	"	"	"	25	81.1	68.7	" "
5.	45	"	"	"	"	1000	85.7	52.7	" "
6.	"	"	"	"	"	25	101.7	68.7	" "
7.	"	"	± 3 Mc/s	2.41	3 Mc/s	1000	62.9	29.9	Television (RMS)
8.	"	"	"	"	"	1000	17.9 (peak)	-15 (peak)	Television (Peak)
9.	"	"	± 100 kc/s	$\sqrt{2}$	≫100 kc/s	1000	72	39	measurement case
10.	"	"	"	"	"	25	88	55	" "
11.	90	"	"	"	"	1000	78	39	" "
12.	"	"	"	"	"	25	94	55	" "
13.	"	"	"	"	5 kc/s	1000	92	53	aural case
14.	"	"	"	"	"	25	108	69	" "
15.	"	F.M.	± 75 kc/s	"	50 µS de-emph. + 8 kc/s L.P. Filt.	1000	120	81	" "
16.	"	"	"	"	"	25	136	97	" "

Referring to column 8, Table 1; we see in lines 1 and 2 the effect on the aural annoyance of a decrease in recurrence frequency of the interference. This decrease is 5.33 octaves and at the rate of 3 db. less annoyance per octave decrease this results in an annoyance diminution of 16 db. Still referring to column 8 and comparing lines 7 and, say, 5 we see the effect of bandwidth. That the decrease in signal to noise ratio is not exactly proportional to the root of the bandwidth ratio $\left(\frac{5\ kc/s}{3\ Mc/s}\right)$ is due to the different response curve shapes in the two cases. The television case with a coupling parameter $n = 2.41$ has a less smooth response curve than the sound case with $n = \sqrt{2}$. Comparing column 8, line 8 with 7 we see a very great difference between R.M.S. and peak signal to noise ratios. Television receivers with their wide bandwidths lead to high crest factors, this particular one being 44.9 db. In columns 7 and 8, lines 9 to 12, the signal to noise ratios are true mean R.M.S. such as would be measured on a thermal meter and not corrected for human ear characteristics. In column 8, lines 13 and 14 should be compared with lines 1 and 2. In the latter case the overall bandwidth is achieved in the I.F. circuits whereas in the former case it is obtained in the audio circuits. The difference in signal to noise ratio caused by this is only 0.3 db. Lines 15 and 16 deal with F.M. The F.M. to A.M. improvement may be seen to be about 28 db.

3. <u>Peak to R.M.S. Ratio or Crest Factor of I.F. stage output.</u>

This ratio can be calculated by means of equation 17, appendix 3 for A.M. and equation 45, appendix 3 for F.M. receivers. This ratio is independent of the radio frequency to which the receiver is tuned, whatever the input waveform, and it will be seen from the formulae to be proportional to the square root of the I.F. bandwidth.

<u>Table 2.</u>

I.F. half bandwidth Δf	n	P.R.F. (f_r)	Crest Factor A.M.	Crest Factor F.M.
± 100 kc/s	$\sqrt{2}$	1000 p/s	28 db	35 db
"	"	25 p/s	44 db	51 db

Table 2 indicates the difficulty in measuring the R.M.S. values without distorting the peaks of the waveform. The ratios quoted in Table 2 are 4 db greater than the ratios obtained from the formulae. This is again in order to correct for the random amplitudes of the interference having a mean value of .636 of the peak value.

4. Influence of I.F. bandwidth on A.F. Output Signal to Noise Ratio.

Figure 12 is a graph of Equation 26, appendix 3 showing the A.F. output R.M.S. signal to noise ratio for an A.M. receiver tuned to 4.2 Mc/s and having an audio bandwidth constituted by a 5 kc/s low pass filter, calculated for various I.F. bandwidths with $n = \sqrt{2}$. Also plotted is the same ratio using equation 27, appendix 3. The 5 kc/s L.P. filter represents a human ear. Both these equations are increased by 4 db. Fig. 13 is a similar graph for an F.M. receiver having an audio stage with 50 µS de-emphasis followed by a low pass filter of 8 kc/s. The 8 kc/s L.P. filter represents the human ear when listening to triangular noise. Both equations are increased by 4 db.

Fig. 14 is the ratio plotted in Fig. 13 for the audio stage with 50 µS de-emphasis and the low pass filter of 8 kc/s, divided by that of Fig. 12. This is, therefore, a plot of the F.M. to A.M. R.M.S. signal to noise ratio improvement ratio. This is for the aural case as 5 kc/s is the equivalent low pass filter for A.M. R.M.S. audio output for the human ear, whilst the 8 kc/s filter is equivalent to the human ear for the triangular spectrum of F.M. From Fig. 14 it may be seen that there is no theoretical limit to the improvement to be obtained by using F.M. There are, however, severe practical limits, amongst them being multi-path distortion, economy of frequency allocations, F.M. improvement threshold, receiver gain and so forth. All these practical limitations mitigate against the use of very wide bandwidths.

Appendix 2.

Comparison of Signal to Noise Ratio formulae with Objective and Subjective Measurements.

1. Measurements of signal to noise ratios for both F.M. and A.M. reception were made using the equipment described in appendix 9. The results of the measurements are tabulated and discussed in this appendix and compared with calculated results from formulae derived in appendix 3.

2. Peak Signal to Noise Ratio.

2.1 I.F. output peak signal to noise ratio.

The following table gives measured ratios of peak signal to peak noise output from an I.F. stage and the corresponding ratios calculated from the formulae in appendix 3 for A.M. (Equation 11) and F.M. (Equation 40).

Table 1.

I.F. Half Bandwidth Δf	n	A.M. Sig/Noise in db.		F.M. Sig/Noise in db.	
		measured	calculated	measured	calculated
$\pm$ 80 kc/s	1.57	36.5	35.8	35.5	31.9
$\pm$ 47 kc/s	1.42	40.5	40	38.0	35.8

It will be seen that the results of A.M. measurements are about $\frac{1}{2}$ db. higher than those calculated from formulae whilst the F.M. measurements are about 3 db. higher than those calculated.

2.2 A.F. output peak signal to noise ratio.

Table 2 below gives the measured peak signal to peak noise ratios from receivers of different I.F. bandwidths and different audio bandwidths where the audio bandwidth was determined by an R and C de-emphasis circuit for both A.M. and F.M.

The formulae for calculating the A.M. ratio is obtained by dividing equation 6 in appendix 3 by the maximum value of equation 21 obtained from the solution of equation 22 and is:

$$\text{A.F. output peak} \left(\frac{\text{Signal}}{\text{Noise}}\right)_{AM} = \pi \frac{f_o}{2f_a} \cdot \frac{n^2}{(1+n^2)\sin\delta} \cdot \frac{1}{\sin\delta e^{-\omega_a t} - e^{-\alpha t}\sin(n\alpha t + \delta)} \quad \text{..........(1)}$$

The equivalent F.M. formula is obtained also from appendix 3 by dividing equation 39 by the maximum value of equation 48 obtained from the solution of equation 49 and is:

$$\text{A.F. output peak}\frac{(\text{Signal})}{(\text{Noise})_{FM}} = \eta\frac{f_o F_1(n)n^2}{2f_a(1+n^2)^{3/2}\sin\delta}\;\frac{1}{\sin(\delta-\delta')e^{-\omega_a t}-e^{-\alpha t}\sin(n\alpha t+\delta-\delta')} \quad (2)$$

An approximate method applicable to cases in which I.F. half bandwidth is greater than audio bandwidth is to calculate the I.F. output peak signal to noise ratio and then to multiply this by the ratio of audio bandwidth to I.F. half bandwidth. This applies both to A.M. and F.M.

Table 2.

I.F. Bandwidth	n	de-emphasis in µS	A.M. Peak $\frac{\text{signal}}{\text{noise}}$ in db.		F.M. Peak $\frac{\text{signal}}{\text{noise}}$ in db	
			measured	calculated	measured	calculated
± 80 kc/s	1.57	25	51.5	50.8	56	58.4
"	"	50	57.5	56.1	62	64.1
"	"	100	63.5	61.6	68	69.9
± 47 kc/s	1.42	25	52.5	51.9	56	58.3
"	"	50	58.0	56.8	62	63.8
"	"	100	64.0	62.3	68	69.6

It will be seen that the calculated and measured results agree to within 2 db. In the case of A.M. the measured ratios are up to 2 db. higher than those calculated, whilst in the F.M. case the measured ratios are down to 2 db. lower than calculated.

2.3 Linearity between Input peak noise to signal ratio and I.F. output peak signal to noise ratio.

Fig. 11 is a plot of measured F.M. peak output signal to noise ratio against peak input noise to signal ratio. This is a straight line for the values of input peak noise to signal ratio sufficiently small to enable the phase deviation angle to remain small enough to be equal to its sine.

3. A.F. output R.M.S. signal to noise ratio.

3.1 A.M.

Table 3 gives the R.M.S. signal to noise ratio for an A.M. receiver and shows the calculated values using equation 27, appendix 3 with 4 db. added. The receiver is tuned to 4.2 Mc/s and the audio stage contains R.C. de-emphasis of various values.

Table 3.

I.F. Bandwidth	n	De-emphasis	Equivalent f_a	Measured signal to noise RMS	Calculated signal to noise RMS
± 80 kc/s	1.57	25 µS	10 kc/s	73 db.	70.5 db.
"	"	50 µS	5 kc/s	76 db.	73.5 db.
"	"	100 µS	2.5 kc/s	79 db.	76.5 db.
± 47 kc/s	1.42	25 µS	10 kc/s	73 db.	70.5 db.
"	"	50 µS	5 kc/s	76 db.	73.5 db.
"	"	100 µS	2.5 kc/s	79 db.	76.5 db.

The measured ratios exceed the calculated by a systematic $2\frac{1}{2}$ db.

3.2 F.M.

Table 4, over, gives the measured and calculated RMS signal to noise ratios for a F.M. receiver tuned to 4.2 Mc/s with the audio stage consisting of R.C. de-emphasis of variable value and followed by a low pass filter. The ratios given in the right hand end column of table 4 are 4 db. greater than the calculated ratios obtained using equation 54 in appendix 3.

Table 4.

I.F. bandwidth	n	de-emphasis R.C.	low pass filter f_c	measured $\frac{\text{signal}}{\text{noise}}$ RMS	calculated $\frac{\text{signal}}{\text{noise}}$ RMS
$\pm$ 80 kc/s	1.57	25 µS	25 kc/s	90.5	90.2
"	"	50 µS	"	96	95.3
"	"	100 µS	"	101	101.0
"	"	25 µS	7 kc/s	99.5	100.0
"	"	50 µS	"	101	103.0
"	"	100 µS	"	104	108.0
$\pm$ 47 kc/s	1.42	25 µS	25 kc/s	85.5	85.2
"	"	50 µS	"	91	90.0
"	"	100 µS	"	96	96.0
"	"	25 µS	7 kc/s	94.5	95.0
"	"	50 µS	"	96	98.0
"	"	100 µS	"	99	103.0

The difference between calculated and measured ratios does not exceed 4 db., and is frequently smaller.

4. F.M. to A.M. Signal to noise ratio improvement

4.1 Both Receivers have Identical Low Pass Filter Audio Circuits.

If we consider two receivers, one A.M. and the other F.M., having audio low pass filters of the same cut off frequency f_a and both having the same I.F. bandwidths wider than about twice the audio bandwidth, i.e. $x < 1$ we can use the formula

$$\frac{\text{Signal}}{\text{noise}} \left(\frac{\text{F.M.}}{\text{A.M.}}\right) \text{RMS} = \sqrt{3} \; \frac{\Delta f}{f_a} \quad \text{......} (3)$$

This is obtained by dividing equation 54 by equation 27 in appendix 3.

Table 5 compares measured and calculated ratios using this formula.

Table 5.

I.F. bandwidth	n	low pass filter cut off f_a	measured $\frac{FM}{AM}$ RMS	calculated $\frac{FM}{AM}$ RMS.
± 80 kc/s	1.57	7.0 kc/s	23.5 db	26 db.
± 47 kc/s	1.42	7.0 kc/s	18.5 db.	21 db.

The calculated values exceed the measured by $2\frac{1}{2}$ db.

4.2 Both Receivers have Identical Audio Circuits consisting of De-emphasis followed by a Low Pass Filter.

In the case where both receivers have the same audio circuit consisting of an R.C. de-emphasis circuit followed by a low pass filter of cut-off f_c we get

$$\text{Signal/Noise} \left(\frac{\text{F.M}}{\text{A.M}}\right) \text{RMS} = \frac{\sqrt{\frac{\pi}{2}}}{\sqrt{\frac{f_c}{f_a} - \arctan \frac{f_c}{f_a}}} \cdot \frac{\Delta f}{f_a} \qquad (4)$$

where $f_a = \dfrac{1}{2\pi RC}$

This is obtained from appendix 3, equations 54, 27, 59 (modified), and the remarks following equation 27.

Table 6 compares measured results with calculation from this formula.

Table 6.

I.F. bandwidth	n	de-emphasis RC	f_c	measured $\frac{FM}{AM}$ RMS	calculated $\frac{FM}{AM}$ RMS
± 80 kc/s	1.57	50 µS	25 kc/s	20 db.	22.0 db
± 80 kc/s	"	100 µS	"	21.5 db.	24.4 db.
± 47 kc/s	1.42	50 µS	"	15 db.	17 db.
± 47 kc/s	"	100 µS	"	16.5 db.	19.4 db.

The errors do not exceed 2.9 db.

5. Aural Subjective Measurements

5. 1 F.M. to A.M. Improvement

It was not found possible to measure by ear the signal to noise ratios of either A.M. or F.M. Several people attempted this but in each case it was found that although the first aural ratio taken was within reasonable agreement with the theoretical, the ear apparently became rapidly tired and the ratio could not be repeated with any agreement.

It was, however, possible to compare the noise from A.M. and F.M. receivers for the same input signal to noise ratio, and this was actually done.

The measurements were carried out using the equipment described in appendix 9. The results of the aurally determined ratios of F.M. to A.M. RMS signal to noise ratios are tabulated in tables 7 and 8 and compared with calculations.

Table 7.

I.F. bandwidth	n	de-emphasis $RC = \frac{1}{2\pi f_a}$	measured $\frac{FM}{AM}$ RMS aural	calculated $\frac{FM}{AM}$ RMS aural
± 80 kc/s	1.57	25 μS	25 db.	26.4
"	"	50 μS	25 db.	26.8
"	"	100 μS	25 db.	29.0
± 47 kc/s	1.42	25 μS	20 db.	21.4
"	"	50 μS	20 db.	20.6
"	"	100 μS	20 db.	24.0

Table 7 compares aurally determined noise ratios with calculations. Both the F.M. and the A.M receiver had identical audio networks consisting of simple de-emphasis circuits. By taking the C.C.I.F. ear response curve and finding its energy bandwidth both for flat and triangular noise spectra it was found that from the annoyance point of view the ear could be considered as a 5 kc/s low pass filter for A.M. flat spectrum noise and as an 8 kc/s low pass filter for F.M. triangular spectrum noise. Thus using equations 54, 27, 59 and 32 from appendix 3 we obtain

Aural Signal/Noise $\frac{(F.M)}{(A.M)}$ RMS $= \frac{\Delta f}{f_a} \sqrt{\dfrac{\arctan \dfrac{5}{f_a}}{\dfrac{8}{f_a} - \arctan \dfrac{8}{f_a}}}$ .... (5)

wherein f_a is the reciprocal of the product of 2π and the de-emphasis time constant in milliseconds, and Δf is in kilocycles per second.

The agreement between theory and subjectivity is seen to be quite reasonable.

A more usual case is that in which the A.M. receiver has no de-emphasis whilst the F.M. receiver has. In this case we get, for our calculations, the expression

Aural Signal/Noise $\frac{(F.M)}{(A.M)}$ RMS $= \dfrac{\sqrt{5}\,\Delta f}{\sqrt{f_a^3 \left(\dfrac{8}{f_a} - \arctan \dfrac{8}{f_a}\right)}}$ .... (6)

This equation is obtained from appendix 3, equations 54, 27 and 59.

Table 8.

I.F. bandwidth	n	De-emphasis used in the FM receiver	Measured $\frac{FM}{AM}$	Calculated $\frac{FM}{AM}$
$\pm$ 80 kc/s	1.57	25 µS	26 db.	25.8
"	"	50 µS	29.0 db.	28.8
"	"	100 µS	32.0 db.	33.4

Table 8 compares aural measurements with expression 6 above. Table 8 shows a normal VHF F.M. receiver compared with a normal VHF A.M. receiver. Here we have very close agreement between calculation and subjectivity. In both cases shown in Tables 7 and 8 the audio bandwidths, apart from de-emphasis were wider than the energy bandwidths of the ear.

5.2 Subjective Effect of De-emphasis

The effect of de-emphasis on annoyance value or apparent loudness of the repeated impulsive interference of random amplitudes was found to increase approximately at the rate of 3 db. per octave of audio bandwidth when a good but not exceptional loudspeaker was used. The bandwidths used were obtained by means of de-emphasis circuits and the limits of bandwidth corresponded with de-emphases of 25 and 100 microseconds. This aurally determined rate of increase of annoyance was the same for A.M. and F.M. systems and is thus in slight disagreement in this respect with theoretical considerations as regards triangular and uniform noise spectra. The rise of annoyance with increasing bandwidth which would be predicted from the theory developed in appendix 3 is shown in Fig.32 to be 4 db. per octave for F.M. and 1.6 db. per octave for A.M. Curve 1, dealing with F.M., was obtained from appendix 3, equations 54 and 59, whilst curve 2, dealing with A.M., was obtained from equations 27 and 32 of the same appendix.

Aural tests were also conducted using a very wide range loudspeaker. In this case the disagreement between theory and practice was greater, in that the rise of annoyance per octave of bandwidth was about 6 db. in both F.M. and A.M. Whether this discrepancy is due in part to the response of the loudspeaker to transient inputs or not is a moot point.

Appendix 3.

Mathematical Theory of Impulsive Interference In A.M. and F.M.

1. Summary

The unit step response of two bandpass coupled tuned circuits as used in a typical receiver I.F. stage will be calculated. The detected outputs both from an A.M. detector and a F.M. discriminator will be passed through a typical audio circuit and the relevant types of signal to noise ratios will be determined. As has been stated previously the effect of the much wider R.F. circuits on the incoming repeated unit steps of interference, though great in itself, is none the less negligible compared with that of the relatively much narrower I.F. circuits. For simplicity only one stage of bandpass coupled circuits is treated, but the influence of more than one stage is discussed in appendix 5.

2. Response of Typical I.F. Stage to Transient Input

Consider Fig. 15, in which the pentode valve is ideal in that its anode-cathode resistance is infinite. The input current is η, and the output voltage, e. Making and combining the "A" matrices of the three impedance groups forming the network, that is the left hand r and C in parallel, the coupled coils L, and the right hand r and C in parallel we obtain

$$\begin{Vmatrix} 1 & 0 \\ \frac{1}{r} + pC & 1 \end{Vmatrix} \times \begin{Vmatrix} \frac{1}{K} & pL(\frac{1}{K} - K) \\ \frac{1}{pLK} & \frac{1}{K} \end{Vmatrix} \times \begin{Vmatrix} 1 & 0 \\ \frac{1}{r} + pC & 1 \end{Vmatrix} \quad \ldots\ldots\ldots\ldots(1)$$

where $p \equiv \frac{d}{dt}$, the differential operator.

The product of the three matrices in equation 1 is

$$\begin{Vmatrix} \frac{1}{K} + pL(\frac{1}{K} - K)\ (\frac{1}{r} + pC) & pL(\frac{1}{K} - K) \\ \frac{1}{K}(\frac{1}{r} + pC) + \frac{1}{pLK} + pL(\frac{1}{K} - K)(\frac{1}{r} + pC)^2 + \frac{1}{K}(\frac{1}{r} + pC) & pL(\frac{1}{K} - K)(\frac{1}{r} + pC) + \frac{1}{K} \end{Vmatrix}$$

$$\equiv \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} \quad \ldots\ldots\ldots\ldots(2)$$

The a_{21} term gives the transfer impedance as

$$\frac{e}{\eta}[p] = \frac{1}{a_{21}} = \frac{pKL}{\left(\frac{pL}{r} + \frac{p^2}{\omega_o^2}\right)\left[2+(1-K^2)\left(\frac{pL}{r} + \frac{p^2}{\omega_o^2}\right)\right] + 1} \qquad (3)$$

where $\omega_o^2 = \frac{1}{LC}$

Now assume $Q >> 1$ and $K << 1$

Let $n = KQ \quad \alpha = \frac{\omega_o}{2Q}$

$$\left.\begin{aligned} p_1 &= \alpha - j(\omega_o - n\alpha) \\ p_2 &= \alpha + j(\omega_o - n\alpha) \\ p_3 &= \alpha - j(\omega_o + n\alpha) \\ p_4 &= \alpha + j(\omega_o + n\alpha) \end{aligned}\right\} \qquad (4)$$

Then equation 3 may be written with adequate approximation

$$\frac{e}{\eta}[p] = Kl\omega_o^4 \frac{p}{(p+p_1)(p+p_2)(p+p_3)(p+p_4)} \qquad (5)$$

3. <u>Considerations dealing with A.M.</u>

3.1 <u>A.M. I.F. Output of Peak Noise and Signal</u>

Now dealing first with the A.M. case, if η represents the amplitude of a steady state sinusoidal current of frequency ω_o, we let $p \equiv j\omega$ and (5) becomes

$$e_1 = \eta Q\omega_o L \frac{n}{1+n^2} \qquad (6)$$

wherein e_1 is the amplitude of the steady state output voltage for an input amplitude of η amperes.

Now to obtain the output voltage $e(t)$ as a time function due to the application of unit step current to the input, we let η equal unit step amperes. By an application of Borel's theorem, and Heaviside's 'shift' theorem equation 5 becomes

$$e(t) = n\alpha Le^{-\alpha t}\left(\cos n\alpha t \sin \omega_o t - \frac{\omega_o}{n\alpha} \sin n\alpha t \cos\omega_o t\right) \qquad (7)$$

As $\omega_o >> n\alpha$ because n is never very far from unity and $Q >> 1$ we may simplify equation 7 by a further approximation to

$$e(t) = -\omega_o L e^{-\alpha t} \sin n\alpha t \cos\omega_o t \qquad (8)$$

The envelope of this function which is pictured in Fig. 6a to d is

$$V_1(t) = -\omega_o L e^{-\alpha t} \sin n\alpha t \qquad (9)$$

where the subscript 1 will refer to A.M., and later, 2 will refer to F.M.

If, during the reception of the repeated unit steps of interference, a steady wanted carrier is present, the envelope, equation 9, will appear as a modulation of the wanted carrier. The greatest value of interference envelope will occur for the in-phase or out of phase conditions of the interference and wanted carrier and it is this condition which we shall treat. The mean value taking all random phase angles into account is calculated in Appendix 6. Thus equation 9 shows the output from the detector. The maximum value of this may be obtained by differentiating equation 9 with respect to time and equating to zero. This process gives

$$\text{Peak } V_1 = \frac{\omega_o L n}{e^{\frac{\delta'}{n}} \sqrt{1 + n^2}} \qquad (10)$$

where $\delta' = \arctan n$

The peak I.F. output signal to noise ratio, assuming 100% modulation of the wanted signal, may be obtained by dividing (6) by (10). Thus

$$\text{I.F. Output Peak } \left(\frac{\text{Signal}}{\text{Noise}}\right)_{\text{A.M.}} = \eta \frac{f_o F_1(n)\ e^{\frac{\delta'}{n}}}{2\Delta f \sqrt{1 + n^2}} \qquad (11)$$

where $F_1(n)$ is a function of n shown in Fig.7 and is obtained from an elementary study of bandpass coupled circuits, and Δf is the half I.F. bandwidth for 3 db. reduction in response from the mid-band value. Examination of Fig.7 shows that

$$\alpha = \frac{\omega_o}{2Q} = \pi \frac{f_o}{Q} = \frac{2\pi\Delta f}{F_1(n)} \qquad (12)$$

3.2 A.M. I.F. Output of R.M.S. Noise and Signal

The R.M.S. noise, for a pulse repetition frequency of f_r, may easily be obtained as follows

$$\overline{V_1^2} = f_r \int_0^{1/f_r} \omega_o^2 L^2 \left(e^{-\alpha t} \sin n\alpha t\right)^2 dt \quad \text{.......... (13)}$$

and if we assume $f_r << \alpha$ we obtain

$$\overline{V_1^2} = \frac{n^2}{1+n^2} \cdot \frac{f_r \omega_o{}^2 L^2}{4\alpha} \quad \text{.......... (14)}$$

Now the signal being sinusoidal has a mean square value of

$$\overline{e_1^2} = \frac{\eta^2 Q^2 \omega_o{}^2 L^2 n^2}{2(1+n^2)^2} \quad \text{.......... (15)}$$

from equation 6.

Thus dividing (15) by (14) and taking the square root gives

$$\text{I.F. Output RMS} \left(\frac{\text{Signal}}{\text{Noise}}\right)_{AM} = \eta \frac{f_o}{\sqrt{f_r \Delta f}} \sqrt{\frac{\pi F_1(n)}{1+n^2}} \quad \text{.......... (16)}$$

3.3 A.M. I.F. Output Noise Crest Factor.

By dividing (10) by the square root of (14) we obtain

$$\text{I.F. Output Noise} \left(\frac{\text{Peak}}{\text{RMS}}\right)_{AM} = 2e^{-\frac{\delta'}{n}} \sqrt{\frac{2\pi \Delta f}{f_r F_1(n)}} \quad \text{.......... (17)}$$

3.4 A.M. A.F. Output of Peak Noise

We now examine the effects produced by an audio circuit upon the I.F. output voltage $V_1(t)$ given by equation 9. Let us assume for simplicity that the audio portion of the receiver consists of a simple de-emphasis circuit of time constant $\frac{1}{\omega_a}$ where ω_a is the angular frequency at which the steady state response is 3 db. less than that obtained for voltages of zero or very low frequencies. The unit step voltage response of the de-emphasis circuit is

$$\frac{\text{Volts out}}{\text{Volts in}}[p]\,1 = \frac{\omega_a}{\omega_a+p} \quad \text{.......... (18)}$$

Now the operational equivalent $V_1[p]$ of $V_1(t)$ is

$$V_1(t) \doteqdot V_1[p] = -\omega_o L \frac{n\alpha p}{(p+\alpha)^2 + n^2\alpha^2} \quad \text{.......... (19)}$$

where $\doteqdot$ means "Operationally equivalent to".

Finally, the operational form of the output from the de-emphasis circuit is

$$V_{1a}[p] = -\omega_o L n\alpha\omega_a \frac{1}{\omega_a+p} \cdot \frac{p}{(p+\alpha)^2 + n^2\alpha^2} \quad \text{.......... (20)}$$

By application of Borel's theorem

$$V_{1a}(t) = -\omega_o L \frac{x\sin\delta}{n}\left[\sin\delta . e^{-\omega_a t} - e^{-\alpha t}\sin(n\alpha t + \delta)\right] \quad \text{.......... (21)}$$

wherein $\delta = \arctan\frac{n}{1-x}$

and $x = \frac{\omega_a}{\alpha} = \frac{f_a}{\Delta f}F_1(n)$

This voltage is pictured in Fig.8.

This maximum value of equation 21 may be obtained by putting into it the value of αt which satisfies the transcendental equation

$$\frac{x\sin\delta}{\sqrt{1+n^2}}\, e^{(\alpha-\omega_a)t} = \sin(n\alpha t + \delta - \delta') \quad \text{.......... (22)}$$

Thus the peak audio signal to noise ratio cannot be given in simple algebraic form. Furthermore, if $\Delta f > f_a$ by a substantial amount, the peak value of $V_{1a}(t)$ is not interesting from an aural point of view because the greater contribution towards it is supplied by the supersonic term $e^{-\alpha t}\sin(n\alpha t + \delta)$. However this peak value is of interest from the noise measurement standpoint and some examples are given in appendix 2, Table 2. Of course if $\Delta f \leqslant f_a$ then the above remarks are not true and the peak value may have an aural interest if the pulse repetition frequency, f_r, is low, as was explained in the main part of this work.

3.5 A.M. A.F. Output of R.M.S. Noise and Signal

Because of this supersonic term in equation 21, it is better from the practical (aural) point of view to calculate the RMS signal to noise ratio by a spectral method which assumes the audio portion of the receiver to consist of an ideal low pass filter. This method results in a formula of equal validity to that which would assume an audio de-emphasis circuit, and is always safe whatever the value of $x = \dfrac{f_a F_1\ (n)}{\Delta f}$. A method based on root mean squaring the time function emerging from a de-emphasis circuit is included in Appendix 4 for interest. This spectral method relies on the following well known identity.

$$f_r \int_0^{1/f_r} [V(t)]^2 dt = \frac{f_r}{\pi} \int_0^{\omega_a} |\phi(j\omega)|^2 \, d\omega \qquad (23)$$

wherein

$$\phi_1(p) = \frac{1}{p} V_1[p]$$

and $V(t)$ is the output voltage emerging from the filter of angular frequency band 0 to ω_a. Thus the mean square audio output voltage emerging from the ideal low pass filter of cut-off frequency $f_a = \dfrac{\omega_a}{2\pi}$ is

$$\overline{V_1{}^2{}_a} = \frac{f_r}{\pi} \int_0^{\omega_a} \left| \frac{\omega_o L n \alpha}{(j\omega + \alpha)^2 + n^2\alpha^2} \right|^2 d\omega \qquad (24)$$

this being evident from an examination of (23) and (19).

$$\overline{V_1{}^2{}_a} = \frac{n f_r f_o{}^2 L^2 x}{4 f_a (1+n^2)} \left[2n \arctan \frac{2x}{(1+n^2) - x^2} + 2.3 \log_{10} \frac{x^2 + 2nx + 1 + n^2}{x^2 - 2nx + 1 + n^2} \right] \qquad (25)$$

Dividing (15) by (25) and taking the square root gives a RMS A.M. audio signal to noise ratio of

AF Output RMS $\left(\frac{\text{Signal}}{\text{Noise}}\right)_{AM}$

$$= \eta\pi f_o \sqrt{\frac{2nx}{(1+n^2)f_r f_\varepsilon}} \cdot \frac{1}{\sqrt{2n\arctan\frac{2x}{(1+n^2)-x^2} + 2.3\log_{10}\frac{x^2+2nx+1+n^2}{x^2-2nx+1+n^2}}} \quad \text{.....(26)}$$

If $x < 1$ equation 26 degenerates to

$$\text{A.F. Output RMS} \left(\frac{\text{Signal}}{\text{Noise}}\right)_{AM} = \eta\frac{\pi}{2} \cdot \frac{f_o}{\sqrt{f_r\ f_a}} \quad \text{.....(27)}$$

for receivers having I.F. bandwidths wider than audio. Now if in fact the audio circuit consists of a de-emphasis network of time constant $\frac{1}{\omega_a}$ then the right hand side of equation 27 must be multiplied by $\sqrt{\frac{2}{\pi}}$ because the energy bandwidth of a de-emphasis network to flat spectrum noise is $\frac{\pi}{2}$ times greater than that of an ideal low pass filter to the same spectrum. In equation 26 we must remember that x becomes $\frac{f^1{}_a F_1(n)}{\Delta f}$ where $f^1{}_a = \frac{\pi}{2} f_a$

3.6 <u>A.M. Aural Output Signal to Noise Ratio.</u>

The actual aural signal to noise ratio with a de-emphasis circuit instead of the low pass filter may also be calculated from equation 26 or 27 but a new special value $f^1{}_a$ of f_a must be calculated to allow for the fact that the ear approximates to a low pass filter having an energy bandwidth to flat spectrum noise of roughly 5 kc/s. A representation of an ear listening to flat spectrum noise emerging from a de-emphasis circuit would thus be a normal de-emphasis response curve cut abruptly at 5 kc/s. The noise energy at the output of such a system would be proportional to

$$N^2_{\text{de-emph}} = \int_0^5 \frac{df}{1 + \left(\frac{f}{f_a}\right)^2} \quad \text{.....(28)}$$

$$N^2_{\text{de-emph}} = f_a \arctan\frac{5}{f_a} \quad \text{.....(29)}$$

whilst the noise energy at the output of the simple low pass filter of cut-off $f^1{}_a$ which we wish to find is proportional to

$$N^{2'}_{L.P.filter} = \int_{0}^{f^1_a} df \quad \text{.......} (30)$$

$$N^{2'}_{L.P.filter} = f^1_a \quad \text{.......} (31)$$

Equating the two noise energies given by (29) and (31) we get

$$f^1_a = f_a \arctan \frac{5}{f_a} \quad \text{.......} (32)$$

Thus the actual value of f_a to be inserted into equations 26 and 27 if a de-emphasis circuit of time constant $\frac{1}{\omega_a}$ is used instead of a low pass filter is f^1_a given by equation 32. It should be remembered that x will now become

$$= \frac{f^1_a}{\Delta f} F_1(n).$$

4. <u>Considerations dealing with F.M.</u>

We now pass to the case of a F.M. receiver possessing a perfect amplitude limiter. The free oscillation set up by the unit step input current will have a constant phase angle with respect to the F.M. carrier assumed unmodulated for simplicity. This assumes that the latter is correctly tuned to the I.F. centre frequency $f_o = \frac{\omega_o}{2\pi}$. This assumption has been shown to be legitimate in practice. We may, therefore, consider only the envelope of this free oscillation, this being given by equation 9. The vector diagram appropriate to this state of affairs is shown in Fig.16.

γ is the constant phase angle between carrier and interference. γ takes random values at each new recurrence of impulsive interference. We shall assume a value of γ which gives a maximum angle of phase shift, β, of resultant from carrier. Actually the value of γ giving maximum rate of change of β varies during the interference envelope cycle but if β is a small angle sufficient accuracy is obtained by letting $\gamma + \beta = \frac{\pi}{2}$ = the third remaining angle. Now

$$\beta = \arcsin \frac{1 + n^2}{\eta nQ} e^{-\alpha t} \sin n\alpha t \quad \text{.......} (33)$$

As n is never far from unity it may be seen that if the carrier to noise ratio is greater than, say, about $\frac{10}{Q}$ (this allows for $n < 2$) β will be a small angle and

$$\beta = \sin \beta \qquad (34)$$

and we can re-write (33) as

$$\beta = \frac{1 + n^2}{\eta \, nQ} e^{-\alpha t} \sin n\alpha t \qquad (35)$$

If the discriminator following the amplitude limiter has a coefficient of proportionality of A volts per cycle, the interference voltage emerging from it will be

$$V_2(t) = \frac{A}{2\pi} \cdot \frac{d\beta}{dt} \qquad (36)$$

$$V_2(t) = \frac{A\alpha(1+n^2)^{3/2}}{2\pi \, \eta \, n \, Q} e^{-\alpha t} \sin (n\alpha t - \delta') \qquad (37)$$

where the subscript 2 refers to F.M. as opposed to A.M. This voltage is pictured after some subsequent selectivity in Fig. 6e and f.

4.1 <u>F.M. I.F. Output of Peak Noise and Signal.</u>

The value of the first peak of $V_2(t)$ occurs, at least mathematically, at time nought and is

$$\text{Peak } V_2 = \frac{A\alpha^2(1+n^2)}{2\pi^2 \, \eta \, f_o} \qquad (38)$$

Selectivity subsequent to the amplitude limiter will, in practice, shift this peak to some time after $t = 0$ but measurements indicate that its value is not greatly altered, certainly not when compared with the alteration incurred after a further passage through the audio circuits which is the case of greater interest. The second and somewhat smaller peak occurs at a time $t = \frac{2\delta'}{n\alpha}$ and may easily be calculated from equation 37. Now assuming 100% modulation of the wanted F.M. signal which we shall define as complete utilisation of the I.F. bandwidth by a deviation of Δf to the -3 db. points in the steady state response curve, the signal voltage emerging from the discriminator is

$$e_2 = A \, \Delta f \qquad (39)$$

whence dividing (39) by (38)

$$\text{IF Output Peak} \left(\frac{\text{Signal}}{\text{Noise}}\right)_{FM} = \eta \, \frac{f_o F_1{}^2(n)}{2(1+n^2)\Delta f} \qquad (40)$$

4.2 F.M. I.F. Output of R.M.S. Noise and Signal

The mean square noise for a pulse repetition frequency of f_r is

$$\overline{V_2{}^2} = f_r \int_0^{1/f_r} \left[\frac{A\alpha(1+n^2)^{\frac{3}{2}}}{2\pi \ \eta \, nQ} \cdot e^{-\alpha t} \sin(n\alpha t - \delta')\right]^2 dt \qquad (41)$$

$$\overline{V_2{}^2} = \left[\frac{A(1+n^2)}{4\pi \ \eta \ Q}\right]^2 \alpha f_r \qquad (42)$$

The mean square signal voltage is, of course,

$$\overline{e_2{}^2} = \frac{A^2\Delta^2 f}{2} \qquad (43)$$

whence, dividing (43) by (42) and taking the square root

$$\text{I.F. Output R.M.S.} \left(\frac{\text{Signal}}{\text{Noise}}\right)_{FM} = \eta \, \frac{f_o}{1+n^2} \sqrt{\frac{\pi F_1{}^3(n)}{f_r \, \Delta f}} \qquad (44)$$

4.3 F.M. I.F. Output Noise Crest Factor

Dividing (38) by the square root of (42) we obtain

$$\text{I.F. Output Noise} \left(\frac{\text{Peak}}{\text{RMS}}\right)_{FM} = 2\sqrt{\frac{2\pi\Delta f}{f_r F_1(n)}} \qquad (45)$$

4.4 F.M. A.F. Output of Peak Noise.

We now examine the effects produced upon the discriminator output, $V_2(t)$, by the audio portion of the F.M. receiver. To obtain the output as a time function we again assume the audio circuits to consist of a de-emphasis network of time constant $\frac{1}{\omega_a}$. Using similar methods and notation to those employed in connection with A.M.

$$V_2(t) \doteqdot V_2[p] = \frac{A\alpha(1+n^2)^{3/2}}{2\pi \ \eta \ Qn} \cdot \frac{pn\alpha\cos\delta' - p(p+\alpha)\sin\delta'}{(p+\alpha)^2 + n^2\alpha^2} \qquad (46)$$

Multiplying (46) by (18) we get for the operational equivalent of the audio noise output voltage

$$V_{2a}\,[p] = \frac{A\alpha(1+n^2)^{\frac{3}{2}}}{2\pi\eta\ Qn}\left[\omega_a \sin\delta'\ \frac{p}{p+\omega_a}\cdot\frac{p}{(p+\alpha)^2+n^2\alpha^2}\right] \qquad (47)$$

By application of Borel's theorem

$$V_{2a}(t) = \frac{A\omega_a^2 \sin\delta\,(1+n^2)^{\frac{3}{2}}}{\pi\,\eta\,n^2\omega_o\,x}\left[\sin(\delta-\delta').e^{-\omega_a t} - e^{-\alpha t}\sin(n\alpha t+\delta-\delta')\right] \qquad (48)$$

This voltage is pictured in Fig. 9.

The maximum value of equation 48 may be obtained by putting into it the value of αt which satisfies the transcendental equation

$$\frac{x\sin(\delta-\delta')}{\sqrt{1+n^2}}\,e^{(\alpha-\omega_a)t} = \sin(n\alpha t+\delta-\delta') \qquad (49)$$

Thus as in the A.M. case the peak audio signal to noise ratio cannot be given in simple algebraic form.

4.5 F.M. A.F. Output of RMS Noise and Signal.

The RMS audio signal to noise ratio is most easily obtained by assuming as we did in the A.M. case that the audio portion of the F.M. receiver consists of an ideal low pass filter of cut-off frequency $\frac{\omega_a}{2\pi}$. Another method is given in Appendix 4 for interest.

Then the mean square noise is

$$\overline{V_2{}^2{}_a} = \frac{f_r}{\pi}\int_0^{\omega_a}\left|\phi_2(j\omega)\right|^2 d\omega \qquad (50)$$

$$\overline{V_2{}^2{}_a} = \frac{f_r}{\pi}\int_0^{\omega_a}\left|\frac{A\alpha(1+n^2)^{\frac{3}{2}}}{2\pi\eta\,Qn}\cdot\frac{n}{\sqrt{1+n^2}}\cdot\frac{j\omega}{j\,(j\omega+\alpha)^2+n^2\alpha^2}\right|^2 d\omega \qquad (51)$$

Finally

$$\overline{V_2{}^2{}_a} = \frac{A^2(1+n^2)^2 f_r f_a^3}{\eta^2 4\pi^2\,n\,f_o^2\,x^3}\left[2n\arctan\frac{2x}{x^2-(1+n^2)} + 2.3\log_{10}\frac{x^2+2nx+1+n^2}{x^2-2nx+1+n^2}\right] \qquad (52)$$

Dividing (43) by (52) and taking the square root gives a RMS F.M. audio signal to noise ratio of

$$\text{AF Output RMS } \frac{(\text{Signal})}{(\text{Noise})_{FM}}$$

$$= \frac{\eta\pi f_o F_1(n)}{1+n^2} \sqrt{\frac{2nx}{f_r f_a}} \cdot \frac{1}{\sqrt{2n \arctan \frac{2x}{(1+n^2)-x^2} - 2.3 \log \frac{x^2+2nx+1+n^2}{10x^2-2nx+1+n^2}}} \qquad (53)$$

If $x < 1$ equation 53 degenerates to

$$\text{A.F.Output RMS } \frac{(\text{Signal})}{(\text{Noise})_{FM}} = \frac{\eta\pi\sqrt{3}}{2} \cdot \frac{f_o \Delta f}{\sqrt{f_r f_a^3}} \qquad (54)$$

for receivers having I.F. bandwidths considerably wider than audio. Now if in fact the audio portion of the receiver actually has a de-emphasis circuit instead of a low pass filter the measured RMS signal to noise ratio might be smaller than that given by equation 53 or 54 by the amount by which the I.F. half bandwidth exceeded the audio band because the F.M. noise spectrum is not flat but proportional to frequency. Thus a de-emphasised F.M.noise spectrum would have the form shown in Fig.17.

4.6 F.M. Aural Output Signal to Noise Ratio.

The reason that the aural signal to noise ratio is not as poor as would be indicated by Fig.17 is that in fact the ear will not pass frequencies above a certain limit and so in effect corresponds with a low pass filter. In fact the energy bandwidth of the ear to triangular spectrum noise is of the order of 8 kc/s so that to obtain the aural A.F. RMS signal to noise ratio in a F.M. receiver, we should use the low pass filter formulae 53 or 54 with a special value f'_a of f_a which we shall proceed to calculate. Consider Fig.18 which shows the effect of a low pass filter of width f'_a and a de-emphasis circuit of time constant $\frac{1}{\omega_a}$ on the triangular noise spectrum.

The curve O C E D represents the de-emphasised noise but restricted in energy bandwidth by the energy band of the ear taken as cutting off at 8 kc/s. If the noise spectrum is taken as reaching an arbitrary value of unity at a frequency Δf, then the de-emphasised noise energy which is heard by the ear is proportional to

$$N^2_{\text{de-emph.}} = \int_0^8 \frac{f^2}{\Delta^2 f (1 + \frac{f}{f_a})^2} \, df = \text{area of OCEDO} \qquad (55)$$

$$N^2_{\text{de-emph.}} = \frac{f_a^3}{\Delta^2 f}\left(\frac{8}{f_a} - \arctan\frac{8}{f_a}\right) \quad \text{.......... (56)}$$

The noise energy emerging from a low pass filter of cut-off f'_a which we require to find is proportional to

$$N^2_{\text{L.P.filter}} = \int_0^{f'_a} \frac{f^2}{\Delta^2 f}\, df = \text{area of OCABO} \quad \text{.......... (57)}$$

$$N^2_{\text{L.P.filter}} = \frac{f'^3_a}{3\Delta^2 f} \quad \text{.......... (58)}$$

Equating these two noise energies we get

$$f'_a = f_a \left[3\left(\frac{8}{f_a} - \arctan\frac{8}{f_a}\right)\right]^{\frac{1}{3}} \quad \text{.......... (59)}$$

Thus the actual value of f_a to be inserted into equations 53 and 54 if a de-emphasis circuit of time constant $\frac{1}{\omega_a}$ is used is f'_a given by (59).

Appendix 4.

Root Mean Square Audio Signal to Noise Ratio in A.M. and F.M. Calculated by a Time Function Method instead of the Spectral Method employed in Appendix 3.

1. A.M.

Let us deal firstly with A.M. Consider Appendix 3 equation 21. It is lengthy but not difficult to calculate the R.M.S. value of $V_{1a}(t)$, but if the frequency $\frac{n\alpha}{2\pi}$ is supersonic the second term in the brackets of this equation will contribute very little to the aural result. If we therefore assume a receiver in which the I.F. half bandwidth is considerably wider than the audio, we neglect this second term and calculate the mean square noise voltage, thus

$$\overline{V_{1a}^{2}} = fr\int_{0}^{1/fr} V_{1a}^{2}(t)\,dt \quad \text{.......... (1)}$$

whence from Appendix 3, equation 21

$$\overline{V_{1}^{2}{}_{a}} = \left(\frac{\omega_o Lx\sin^2\delta\cdot\sqrt{f_r}}{n}\right)^2 \int_{0}^{1/fr} e^{-2\omega_a t}\,dt \quad \text{.......... (2)}$$

which if $f_r << \omega_a$ (normally the case) becomes

$$\overline{V_{1}^{2}{}_{a}} = \frac{1}{2\omega a}\left(\frac{\omega_o Lx\sqrt{f_r}\,\sin^2\delta}{n}\right)^2 \quad \text{.......... (3)}$$

Dividing equation 15, Appendix 3 by equation 3 above and taking the square root

$$\sqrt{\frac{\overline{e_1^{2}}}{\overline{V_{1a}^{2}}}} = \eta\sqrt{\frac{\pi}{2}}\,\frac{n^2 f_o}{(1+n^2)\sqrt{f_r f_a}\,\sin^2\delta} \quad \text{.......... (4)}$$

But as we have assumed that the I.F. half bandwidth is considerably wider than audio we have implicitly taken $x << 1$ so we must find the limit of (4) for $x \longrightarrow 0$. This process results in

$$\text{A.F. Output RMS}\ \frac{(\text{Signal})}{(\text{Noise})_{AM}} = \eta\sqrt{\frac{\pi}{2}}\,\frac{f_o}{\sqrt{f_r\,f_a}} \quad \text{.......... (5)}$$

This formula becomes the same as equation 27, Appendix 3 if we remember that the energy width of our ideal low pass filter which would pass the same energy as the de-emphasis circuit of time constant $\frac{1}{\omega_a} = \frac{1}{2\pi f_a}$ is

$$f_a^{1} = \frac{\pi}{2} f_a \quad \text{.......................} (6)$$

Thus, expression 5 in terms of f_a^1 becomes expression 27 in Appendix 3.

2. F.M.

Applying precisely the same methods to equation 48, Appendix 3 we get

$$\text{A.F. Output RMS} \left(\frac{\text{Signal}}{\text{Noise}}\right)_{\text{F.M.}} = \eta \sqrt{\frac{\pi}{2}} \frac{f_o \Delta f}{(1+n^2)\sqrt{f_r f_a^3}} \quad \text{.......................} (7)$$

This equation is similar to equation 54, Appendix 3.

By equating the two expressions we see that the effect of neglecting the supersonic term in equation 48, appendix 3, is equivalent to the insertion in the circuit of a low pass filter of cut-off frequency equal to

$$f_a \quad \sqrt[3]{\frac{3}{2}\pi\,(1+n^2)^2}$$

If the I.F. circuit coupling has the optimum or critical value corresponding with n=1, this cut-off frequency becomes 2.66 f_a.

Appendix 5.

Response to Unit Step of Two Band Pass Coupled Transformer Networks in Cascade with Ideal Pentode Valve between them.

The theory outlined in Appendix 3 is restricted to a single band pass coupled system. It would be desirable to know whether the results of that theory still hold reasonably good when more than one such coupled system is employed. We therefore examine the indicial transfer impedance of the system pictured in Fig.19.

The pentode mutual conductance is S amperes per volt. From appendix 3, equation 5, we have

$$Z[p] = KL\omega_o^4 \frac{p}{(p+p_1)(p+p_2)(p+p_3)(p+p_4)} \qquad (1)$$

where $Z[p]$ is the indicial transfer impedance of one of the boxes in Fig.19.

Now
$$e_1 = Zi_1 \qquad (2)$$
$$e_2 = ZSe_1 \qquad (3)$$

whence

$$e_2 = SZ^2 i_1 \qquad (4)$$

and operationally

$$e_2(t) \doteqdot e_2[p] = S.Z^2[p] \qquad (5)$$

wherein i_1 = unit step.

By application of Borel's theorem

$$e_2(t) \doteqdot \frac{1}{p} Z[p] \;.\; pZ[p].S \qquad (6)$$

and

$$e_2(t) = S\int_o^t f_1(u).f_2(t-u)du \qquad (7)$$

with

$$\left.\begin{aligned} f_1(t) &\doteqdot Z[p] \\ f_2(t) &\doteqdot pZ[p] \end{aligned}\right) \qquad (8)$$

Now from Appendix 3, equation 8

$$f_1(t) = -\omega_o L e^{-\alpha t} \sin n\alpha t \cos \omega_o t \quad \text{.......... (9)}$$

From the second equation 8 above it is clear that

$$f_2(t) = \frac{d}{dt} f_1(t) \quad \text{.......... (10)}$$

whence

$$f_2(t) = \frac{\omega_o L}{2} e^{-\alpha t} \quad [\alpha \sin(\omega_o + n\alpha)t - \alpha \sin(\omega_o - n\alpha)t$$

$$- (\omega_o + n\alpha)\cos(\omega_o + n\alpha)t + (\omega_o - n\alpha)\cos(\omega_o - n\alpha)t] \quad \text{.......... (11)}$$

Forming $f_1(u) \cdot f_2(t-u)$, then effecting the integration indicated in (7), and finally making the approximations permitted by the assumption that $\omega_o >> n\alpha$ we get

$$e_2(t) = -S\frac{\omega_o^3 L^2}{8n\alpha} e^{-\alpha t}[(2\sin n\alpha t - n\alpha t \cos n\alpha t)\sin\omega_o t - n\alpha t \sin n\alpha t$$

$$\cos \omega_o t] \quad \text{.......... (12)}$$

The envelope of this function is

$$\text{Envelope of } e_2(t) = -S\frac{\omega_o^3 L^2}{8n\alpha} e^{-\frac{y}{n}} \sqrt{y^2 + 4\sin^2 y - 2y \sin 2y} \quad \text{.......... (13)}$$

wherein $y = n\alpha t$

From (9) it is evident that the envelope of the output voltage from a single bandpass coupled system is

$$\text{Envelope of } e_1(t) = -\omega_o L e^{-\frac{y}{n}} \sin y \quad \text{.......... (14)}$$

We shall now examine two cases. First let us compare the two envelopes (13) and (14) when the overall steady state transfer impedance of the double bandpass coupled system shown in Fig.19 is made equal to that of one of the single systems which go to form it.

From Appendix 3, equation 6

$$\frac{e_1}{i_1} = \frac{Q\omega_o L n}{1+n^2} \quad \text{.......... (15)}$$

so that

$$\frac{e_2}{i_1} = \left(\frac{Q\omega_o Ln}{1+n^2}\right)^2 S \qquad (16)$$

and for

$$\frac{e_2}{i_1} = \frac{e_1}{i_1} \qquad (17)$$

we must have

$$S = \frac{1+n^2}{Q\omega_o Ln} \qquad (18)$$

Putting this value of S into (13) we get

$$\text{Envelope of } e_2(t) = -\frac{\omega_o L(1+n^2)}{4n^2} e^{-\frac{y}{n}} \sqrt{y^2+4\sin^2 y - 2y\sin 2y} \qquad (19)$$

Fig.20 is a plot of equations 14 and 19. It may be seen that the cascading of two identical coupled circuit systems reduces the amplitude of the unit step response by 1.4 db. below that of a single coupled circuit system whilst at the same time requiring 1.6 times the time to reach this maximum.

Secondly, we shall compare the unit step response of a single coupled circuit system having a given bandwidth with a double cascaded coupled circuit system having the same overall bandwidth. This is a more interesting case than the preceding one. As with Fig.20 we shall let n = 1 in both cases so that the general shape of the double system will be similar to that of the single system. Now if at a certain off-tune frequency the single system is, say, 3 db. down in steady state response, then for equivalent bandwidth we must arrange that for the same off tune frequency each of the two circuits of the double system must be $1\frac{1}{2}$ db. down. An elementary study of coupled circuits will then show that the Q factor of each circuit of the double system must be eight tenths of the Q factor of the single system. Bearing this in mind and letting Q be the Q factor of the single system and 0.8Q be that of the double system we find that whilst the co-efficient of equation 14 is unaltered that of equation 19 becomes $\frac{\omega_o L(1+n^2)}{3.2\,n^2}$ because Q in equation 18 becomes 0.8Q. With n=1 this factor becomes 0.625 $\omega_o L$ whereas before, in Fig.20 it was 0.5$\omega_o L$. Thus we re-plot curve 1, Fig.20 on Fig.21, and curve 2 is re-plotted on this same figure but with its ordinates multiplied by $\frac{0.625}{0.5} = 1.25$ and its abscissae multiplied by 0.8. Thus we see that for more or less equivalent overall bandwidths the double cascaded scheme is scarcely different either in amplitude or in response time from the single system.

The conclusions from the theory expounded in Appendix 3 may therefore be applied with good accuracy to receivers having two I.F. bandpass coupled circuit systems in cascade. It would not appear unreasonable to assume that a greater number of such systems in cascade would scarcely falsify our conclusions provided the overall bandwidths were kept constant.

Appendix 6.

The Mean Value of Repeated Pulses having Random Amplitudes.

1. The Mean Value of the Resultant.

The signal to noise ratios calculated in Appendix 3 were all based on the maximum noise condition due to the assumption that the steady carrier either in A.M. or F.M. was in phase or in opposition to the I.F. oscillation of the noise pulse. In fact, however, all phase angles will arise during reception of continually repeated impulsive interference and we now turn to the calculation of the mean noise. Consider, Fig.22, a carrier vector of amplitude $C > 1$ which with a noise vector of unit amplitude forms a resultant R which we shall investigate. This resultant, R, is

$$R = \sqrt{1 + C^2 + 2C\cos\theta} \qquad \text{(1)}$$

The mean value of all the different resultants obtained when θ takes all possible angles between 0 and infinity radians is, by symmetry

$$\overline{R} = \frac{1}{\pi}\int_0^{\pi} R\,d\theta \qquad \text{(2)}$$

or

$$\overline{R} = \frac{2}{\pi}(1+C).E\left(\frac{\pi}{2}, \frac{2\sqrt{C}}{1+C}\right) \qquad \text{(3)}$$

where E is the complete elliptic integral of the second kind. $\overline{R} - C$ is shown in Fig.23. Equation 3 has a certain interest for the measurement of noise by means of a detector in the output from the I.F. circuits of a noise measuring receiver. If this detector measures the mean value of the noise pulses while remaining a peak detector to voltages of intermediate frequency it will indicate the quantity

$$C(1-f_r W) + \overline{R}\, f_r W$$

where f_r is the P.R.F. and W the width of each pulse assumed rectangular for simplicity.

2. The Mean Value of the Detected Resultant.

Let us now imagine that the I.F. detector has a time constant long compared with the I.F. period but short compared with the I.F. half bandwidth frequency - that is, we assume that the noise pulses are passed on to the receiver audio or video circuits without distortion. Now let us measure these pulses with a mean rectifier. It will differ in its indication depending upon whether it reads positive or negative going pulses, Fig.24. This figure is simply a graph of the arithmetic difference R - C for all phase angles between carrier and interference vectors between 0 and π. If the meter read positive going pulses, for example, its reading would be proportional to the mean value taken over the phase angle interval 0 to π of all vectors greater than the carrier, C. The level of I.F. detector operation is clearly set by the mean I.F. voltage $\bar{R}$. The angle θ' at which the resultant R becomes equal to the carrier C is

$$\theta' = \pi - \arccos \frac{1}{2C} \quad \text{.......... (4)}$$

Thus if M_p and M_n are the mean meter readings for positive and negative going pulses respectively we have

$$M_p = \frac{1}{\pi} \int_0^{\theta'} (R - C)d\theta \quad \text{.......... (5)}$$

and as

$$\bar{R} = \frac{1}{\pi} \int_0^{\theta'} Rd\theta + \frac{1}{\pi} \int_{\theta'}^{\pi} Rd\theta \quad \text{.......... (6)}$$

or

$$\bar{R} = M_p - M_n + C \quad \text{.......... (7)}$$

$$M_n = C + M_p - \bar{R} \quad \text{.......... (8)}$$

Now from (5)

$$M_p = \frac{2(1+C)}{\pi} E\left(\frac{\theta'}{2}, \frac{2\sqrt{C}}{1+C}\right) - \frac{\theta'}{\pi} C \quad \text{.......... (9)}$$

Equations 8 and 9 are plotted in Fig.25. The two curves shown take no account of pulse width W or repetition frequency f_r. Thus they should be multiplied by $f_r W$ to obtain the actual mean values for a carrier to noise ratio of $C > 1$. If an audio RMS signal to noise ratio be measured then both positive and negative peaks would contribute and the mean RMS signal to noise ratio taken over many successive repetitions of impulsive interference would be proportional to the sum $M_p + M_n$ which for carrier to noise ratios of $C > 1$ is about 0.62. When C increases indefinitely $M_p + M_n$ tends to the mean value of a half sinusoid, namely 0.636. This means that all the RMS signal to noise ratios given in Appendix 3 should be increased by about 4 db. It is assumed that the aural results would follow the same trend in that the ear would tend to average the various random amplitudes and give an impression proportional to $M_p + M_n = 0.62$.

Appendix 7.

Use of Unit Impulse instead of Unit Step.

Consider Appendix 3, equation 9. If we assume the input noise current to be a unit impulse of value U coulombs, where

$$U = B\tau \quad \text{.......} \quad (1)$$

B being evaluated in amperes and τ in seconds

then the envelope of the unit impulse response corresponding with the unit step response $V_1(t)$ of equation 9, Appendix 3 is

$$V_1^*(t) = U \frac{d}{dt} e(t) \quad \text{.......} \quad (2)$$

or

$$V_1^*(t) = -U\omega_o^2 L e^{-\alpha t} \sin n\alpha t \quad \text{.......} \quad (3)$$

For the two responses $V_1(t)$ and $V_1^*(t)$ to be identical we must have

$$U = \frac{1}{\omega_o} \quad \text{.......} \quad (4)$$

Thus to convert all the signal to noise ratio formulae to a unit impulse response it is only necessary to divide them by $\omega_o = 2\pi f_o$. In this case we must make

$$B\tau = \frac{1}{\omega_o} \quad \text{.......} \quad (5)$$

Appendix 8.

Influence of the Charge and Discharge Time Constants of a Noise Measuring Detector upon the Meter Indication

The international standard noise measuring receiver has its indicating detector at the output from the I.F. circuits, though an A.F. detector is permitted if its readings accord with those of the I.F. detector for the same interference. The charge and discharge time constants so far adopted as standard are one, and one hundred and sixty milliseconds respectively, though recently a discharge time of five hundred milliseconds has been proposed. The relatively long charge time coupled with the standardised receiver bandwidth of $\pm 4\frac{1}{2}$ kc/s results in a somewhat high peak to mean detector current ratio and this becomes difficult to measure when the P.R.F. of the interference is low. To become familiar with the mode of operation of such a detector consider Fig.26. The interference is taken as pulses of unit height and W width repeated at a P.R.F. of f_r. We assume the pulse periodicity $\frac{1}{f_r} \gg$ W, the width. We also take a discharge time much greater than the charge time, $T_d \gg T_c$.

We may neglect the small reduction in charge due to the fact that even while charging, the meter is also discharging. This negligible reduction is shown in Fig.26 as "a".

After a certain time depending on pulse width, P.R.F. and charge and discharge times, the meter through which is passing the mean detected current will settle down to a fixed value which will be the mean of the current cycle DGH when the points D and H have the same heights. This state of affairs will occur when the amount of charge given by each pulse equals the amount of discharge during the quiescent period between successive pulses. If A is the detected current at the point in the cycle DGH shown by D, then the current supplied by the pulse will be 1-A. The current B at the point G will be

$$B = (1-A)(1-e^{-W/T_c}) + A = 1-(1-A)e^{-W/T_c} \quad \ldots\ldots (1)$$

The current at the point H will be

$$A' = \left[1 - (1-A)e^{-W/Tc}\right] e^{-(1/f_r - W)/Td} \quad \ldots\ldots (2)$$

When a steady condition is attained, the points D and H will be at the same height so we may write

$$A' = A \qquad (3)$$

whence

$$A = \frac{(1-e^{-W/Tc})\, e^{-(1/f_r - W)/T_d}}{1-e^{-[(1/f_r-W)/T_d + W/Tc]}} \qquad (4)$$

The mean detected current as indicated by the D.C. meter will clearly be

$$J = f_r \left\{ \int_0^W [1 - (1-A)e^{-t/Tc}]\,dt + \int_0^{1/f_r - W} Be^{-t/T_d}\,dt \right\} \qquad (5)$$

For simplicity let

$$\mu = W/Tc \qquad \nu = W/Td \text{ and } \sigma = 1/(T_d . f_r) \qquad (6)$$

Equation 5 becomes

$$J = Wf_r[1 + (\tfrac{1}{\nu} - \tfrac{1}{\mu}) \frac{(1-e^{-\mu})(1-e^{\nu-\sigma})}{1-e^{\nu-\mu-\sigma}}] \qquad (7)$$

If the I.F. bandwidth preceding this indicating detector is less than twice the audio width preceding it

$$W = 1/(2\Delta f) \qquad (8)$$

If the audio is narrower than the I.F. half bandwidth then

$$W = 1/(\text{twice audio width}) \qquad (9)$$

Figure 28 shows how the meter behaves for various circuit bandwidths, and hence pulse widths. The curves are direct applications of equation 7 and are based on the pulse height which would result from a receiver bandwidth of $\pm$ 5 kc/s. This means that the zero decibel line corresponds with Δf = 5 kc/s and the curves shew mean meter readings for various bandwidths in decibels above or below the meter reading for the $\pm$ 5 kc/s. bandwidth.

It may be noted that if both charge and discharge time constants be multiplied by the same number, the ratio being kept constant, the mean meter readings will not change; thus compare the two sets of conditions giving rise to the same curve A. The effect of increasing the discharge time constant only, thus increasing the ratio of discharge to charge time constants may be seen by comparing curves A and C or curves B and D. Fig. 27, also a direct application of equation 7, is of much more interest as noise meters do not normally have variable bandwidths but do have to measure impulsive noises of various repetition frequencies. A careful scrutiny of the curves shown in Fig. 27 reveals the following points:-

(a) The shape of the curve of mean detected current plotted against P.R.F. is a constant whatever the time constants used for the charge and discharge circuits.

(b) Multiplying the discharge time constant or dividing the charge time constant by a number, N, shifts the entire curve N times downwards along the P.R.F. axis towards lower pulse repetition frequencies.

(c) Multiplying both time constants by a number, N, does not change the curve in any way. However, it has the important effect of making the meter needle read steadily for low P.R.F.

(d) For a noise meter having a ratio of discharge to charge time constants of 160, the portion of the curve tangent to a curve having a RMS characteristic of 3 db/octave of P.R.F. is centred around a P.R.F. of 60 p/s.

(e) If, for example, it is required to design a noise meter having the 3 db/octave portion around a P.R.F. of F pulses per second, the ratio of discharge to charge time constants must be

$$\frac{Td}{Tc} = 160 \quad \frac{60}{F} = \frac{9600}{F}$$

(f) the region, delineated by pulse repetition frequencies F_1 and F_2, over which the curve does not depart seriously from the slope of 3 db/octave can be shown in the following table.

Departure from RMS = 3 db/octave	F_2/F_1
$^{+0}_{-1}$ db	7.9
$^{+0}_{-2}$ db	19
$^{+0}_{-3}$ db	37

Appendix 9.

Experimental Equipment and Procedure.

1. Description of Apparatus.

1.1 Unit Step Generator (Fig.30)

This comprises pulse and time base generator circuits. The condenser C_1 is charged through the resistance R_3 from the H.T. line during the quiescent period of the thyratron V_3, and discharges through the thyratron when it strikes, producing across the cathode resistor a voltage waveform approximating to unit step, rising very rapidly and decaying according to the discharge circuit time constant.

The time base generator is similar, with a condenser discharging through the thyratron V_2 and a pentode valve V_1 to ensure linearity.

Provision is made for control of the time base speed by means of the potentiometer VR_1 and the phase of the output unit step can be varied relative to the time base by means of VR_2.

1.2 Square-Wave Generator (Fig.29)

This provides the triggering impulse to the two thyratrons and also a positive going brilliance/black-out square wave for the cathode ray oscilloscope. Valves V_1 and V_2 form a multivibrator circuit and provision is made for rapid switching between any two values of pulse repetition frequency selected by pre-set adjustments of the potentiometers VR_1 and VR_2. The range of P.R.F. obtainable is approximately 10-1000 p/s.

1.3 A.M./F.M. Receiver (Fig.1)

1.3.1 Tuned circuits under Test (See Response Curves Fig.5)

The coupling between valves V_1 and V_2 consists of two switchable pairs of coupled circuits of nominal bandwidths $\pm$ 47 kc/s and $\pm$ 80 kc/s and coupling parameters 1.42 and 1.57 respectively. The upper limit of bandwidth was set by the detector time constants required to follow the frequencies of the resultant detected envelopes and the lower limit by the Q of the coils available. The mid-frequency of both of these circuits is 4.2 Mc/s.

1.3.2 A.M.Circuits.

When used for A.M., two further I.F. stages are used following the coupled circuits, and an infinite impedance detector V_7, this latter being employed in order to obtain a low output impedance without sacrificing linearity at low input voltages.

1.3.3 F.M. Circuits.

In the F.M. case, the valve V_3 is followed by two limiter stages and a conventional Foster-Seeley type balanced phase discriminator having a bandwidth slightly more than double that of the wider coupled circuits. It will be noticed that resistive couplings are used between V_3 and V_4 and V_4 and V_5, as it was found that with impedances which were not aperiodic the rate of rise of voltage across the circuit due to the initial excursion of the limiters into grid current was sufficiently rapid to produce a "ring".

1.3.4 Audio Circuits.

These comprise V_8, a pentode amplifier stage with four values of switchable de-emphasis, 0, 25, 50 and 100 microseconds followed by V_9, a cathode follower.

2. Objective Experiments.

2.1 A.M. Peak Signal to Noise ratio.

The output of the unit step generator, and an unmodulated carrier of frequency 4.2 Mc/s and amplitude lying within the range of linearity of the receiver, were applied simultaneously to the receiver. The pulse or unit step was adjusted to a value which made the peak amplitude of the pulse output envelope from the coupled circuits not greater than the carrier amplitude at the same point. The receiver output in these conditions, on both bandwidths and with various values of de-emphasis was examined on the cathode ray oscilloscope (see Figs. 6c and d and 8a to f) and their peak amplitudes measured. It can be seen from these photographs that amplitudes of successive pulses are completely random, and depend on the relative phase of impulse and carrier, being at a maximum in one polarity when both are in phase and maximum in the opposite polarity when in phase opposition.

It may be mentioned in passing that the P.R.F. used for these photographs was 1,000 p/s and the exposures given were of the order of $\frac{1}{2}$ second.

Having measured the peak output pulse amplitude from the receiver, the pulse input was also measured, and then the receiver output, with the pulse removed and the carrier at the same strength but 100% modulated. From this data the signal to noise ratios, both input and output, were calculated and combined to obtain an overall signal to noise ratio. These results are tabulated in Appendix 2, Tables 1 and 2 and compared with those obtained by calculation from the formulae derived in Appendix 3.

2. 2 F.M. Peak Signal to Noise Ratio.

A similar routine to that in the previous experiment was adopted, measurements being made with various amplitudes of carrier and input signal to noise ratios, within the limits prescribed by the carrier being well above limiting level and the input signal to noise ratio sufficiently high to prevent the R.F. pulses in antiphase with the carrier, from dropping below limiter level and producing an A.M. component in the output. The C.R.O. pictures under these conditions are shown in Figs. 6e and f and 9a to f and the results tabulated and compared in Appendix 2, Tables 1 and 2.

2. 3 A.M. RMS Signal to Noise Ratio.

The layout of apparatus was as shown in Fig. 31, both the audio amplifiers being BBC type MPA/1. These have flat responses from 30 c/s to 25 kc/s and power outputs of 10 watts. A vacuo-junction coupled to a 0 to 30 microammeter was used as the square-law measuring device.

The procedure adopted was to apply a 25% modulated carrier to the receiver and note the A.F. attenuation required to give a certain reading on the meter. The modulation was then removed, but retaining the same carrier amplitude a measured amplitude of unit step was applied. The attenuation required to give standard reading on the meter was noted. The difference in the attenuator readings was thus the output signal to noise ratio for 25% modulation. The input signal to noise ratio being known, the overall ratio was calculated for 100% modulation by adding the appropriate correction factor.

It was not possible to make this measurement with zero de-emphasis, as the H.F. response of the audio amplifiers did not extend above 25 kc/s, neither could they cope with the high crest factor of the output wave-form under no de-emphasis conditions. Measurements were, however, taken with the three values of de-emphasis in the receiver, also with 3.5 kc/s and 7 kc/s low pass filters in addition to de-emphasis. The results obtained are tabulated and compared with the calculated values in Appendix 2, Table 3.

2.4 F.M. RMS signal to noise ratio.

The procedure was precisely as in the A.M. case. Results are tabulated in Appendix 2, Table 4.

3. Subjective Experiments.

The layout of apparatus was as in Fig. 31.

3.1 Relative annoyance of repeated impulses of uniform amplitude. Effect of P.R.F.

In this experiment the output of the unit step generator was fed into the receiver unaccompanied by carrier, giving at the loudspeaker a succession of impulses of uniform amplitude. A musical programme was also fed to the amplifier and a number of observers were asked to estimate the annoyance level of the interference.

With each observer the P.R.F. was altered in steps from 10 p/s to 1000 p/s, the amplitude being maintained constant, and the attenuation altered at each step until in the opinion of the observer the annoyance level was the same. Curves were plotted of these values of attenuation against P.R.F. and the average of these curves is given in Fig.4, compared with the energy curve of 3 db. per octave.

3.2 Relative annoyance of repeated impulses of random amplitudes. Effect of P.R.F.

In this experiment the impulses were again fed through the receiver this time in the presence of a carrier, thus giving an output of random amplitude. Apart from this the procedure was as detailed in section 3.1. The average curves are given in Fig.3. For curve "A" the annoyance levels of various values of P.R.F. were compared with that of 25 p/s and the attenuation adjusted to give equality of annoyance. In curve "B" the attenuation at each P.R.F. was adjusted to give a "just disturbing" interference level.

3.3 Effect of I.F. bandwidth on Annoyance of interference. A.M. and F.M.

Using the apparatus of Fig.31 with the low-pass filters and programme source out of circuit, the effect of switching receiver I.F. bandwidths was investigated.. It was found that with all three values of de-emphasis there was no effect on the annoyance of interference, either on F.M. or A.M. This was because in all three cases the I.F. half width being greater than audio, the interference was determined solely by the latter.

3.4 Effect of de-emphasis on annoyance of interference. A.M. and F.M.

The experimental method was the same as in section 3.3. The value of de-emphasis was switched from 25 to 50 and then 100 μS and at each point the attenuation in the audio chain was adjusted to equalise the annoyance.

This was repeated on A.M. and F.M. The results are given in Appendix 2, paragraph 5.2.

3.5 Ratio of Annoyance of interference. FM/AM.

The experimental arrangements were as described in section 3.3. The relative gains of the F.M. and A.M. channels of the receiver were adjusted so that at a predetermined carrier level the audio outputs, for the same depth of modulation (or deviation) were equal.

The effect of switching from A.M. to F.M., with the input unit steps superimposed on this level of carrier, was noted, the attenuation being adjusted for equal annoyance as in section 3.4.

This was repeated with each value of de-emphasis and the results obtained are given in Appendix 2, Tables 7 and 8.

Appendix 10.

F.M./A.M. Improvement Threshold. (Pops and Clicks).

1. Interference can be heard as Clicks even when I.F. output noise peak is greater than Carrier.

All the foregoing analysis and discussion relating to F.M. only apply when the noise peak at the limiter input is somewhat less than the carrier amplitude, in fact, sufficiently less for β to equal $\sin\beta$ and $\tan\beta$, Fig. 16. We shall now discuss in a qualitative manner what happens when this condition does not obtain. Rigid mathematical analysis can only be used if solutions in terms of infinite series are employed, and even then these are far from simple and only valid within strict limits. Let us visualise the state of affairs occurring when the noise peak exceeds the carrier. Fig. 33 shows what happens when the random phase angle, γ, is $\frac{\pi}{2}$ radians. For values of $\gamma > \frac{\pi}{2}$ the instantaneous discriminator output voltage, $\frac{A}{2\pi} \cdot \frac{d\beta}{dt}$ will approach the form of curve already dealt with and pictured in Fig. 6e and f. Now consider a very small angle γ, Fig. 34. If the limiter circle had had a smaller radius the instantaneous discriminator output voltage would have followed the dotted curve, but as this output voltage is proportional, not only to the time derivative of the phase deviation, but also to the limiter output voltage, it suffers a decrease of amplitude as the operating point of the vector CD passes between A and B. Though the points A and B will always be at the intersection of the noise vector and the limiter circle, they are not necessarily symmetrically disposed around the maxima of the output voltage curve shown at the right of Fig. 34. Their position will depend upon the relative amplitude, P, of the noise vector and upon the particular value which the random angle γ may have. It may also happen that the rate of change of β may be so great that the maxima, H, exceed the frequency width of the discriminator. In this case the actual output voltage may fall momentarily whilst the frequency deviation due to the noise vector is beyond the outer cut-off points of the discriminator. A little thought will show that provided - and this is important - the limiter time constant is really short and the discriminator is really symmetrical about the I.F. mid-band frequency, none of these effects described above are of over-riding importance in that they do not change the triangular shape of the audio spectrum due to the more or less symmetrical bi-polar pulse which may be regarded as the time derivative of the unit impulse and is typical of correctly functioning F.M. reception.

Those familiar with F.M. reception will know that when the signal to noise ratio in a F.M. receiver is so poor that the F.M./A.M. advantage decreases and departs seriously from the theoretical value (about 26db. for a 75 kc/s deviation and 50 μS. pre and de-emphasis system) it is because the audio output noise begins to sound similar to what it would be like in an A.M. system. Instead of the pulses of interference sounding high pitched like clicks (triangular spectrum) they sound as if there is more bass in them, like pops (uniform spectrum).

In an ideal F.M. receiver there are two possible causes for the occurrence of pops as distinct from clicks. One cause can produce pops without the carrier being modulated and the other is due to the modulation. The latter case has been thoroughly covered by Smith and Bradley in Proceedings of the Institute of Radio Engineers, for October 1946.

2. Mechanism of the Production of Pops.

To understand the first cause, let us revert to the complete expression for the I.F. noise output given in appendix 3, equation 7. This may also be written

$$e(t) = n\alpha Le^{-\alpha t} \sqrt{\cos^2 n\alpha t + \frac{4Q^2}{n^2} \sin^2 n\alpha t} \cdot \sin\left[\omega_0 t - \arctan \left(\frac{2Q}{n} \tan n\alpha t\right) \right] \qquad (1)$$

or approximately

$$e(t) = \omega_0 Le^{-\alpha t} \sin n\alpha t \cdot \sin\left[\omega_0 t - \arctan \left(\frac{2Q}{n} \tan n\alpha t\right)\right] \qquad (2)$$

This expression 2 shows that in the simplifying assumption which led to equation 8, appendix 3, we omitted to account for the effects which might result from the phase angle $\psi = \arctan \frac{2Q}{n} \tan n\alpha t)$. This variable phase angle imposes upon the noise vector a rotation of $-\frac{\pi}{2}$ radians for each $\frac{\pi}{2}$ increase in $n\alpha t$, Fig.35. Fig. 36 shows three curves of the variation of the instantaneous amplitude of the noise vector for three values of the relative amplitude P. P is of course the ratio

$$P = \frac{1 + n^2}{n\, Q\, \eta} \qquad (3)$$

as may be seen from inspection of Fig.16. Fig.36 is a theoretical graph of the same function as shown in the actual photographs of Fig.6c and d. If now the vector diagrams as shown in Figs.16, 33 and 34 be replotted taking into account the phase angle ψ, we get the three diagrams of Fig.37, reading from left to right.

In the three cases shown in Fig.37 the angle β starts at zero and finishes at zero, but a small change in the random starting phase angle γ would, in Fig.37b, have caused the resultant vector to start, as before, at zero but to finish at -2π radians. This event would have occurred if the point 0 had been inside the closed oval shaped operating locus instead of just outside it as actually shown. Fig.38 shows the angle β plotted against time, or rather against $n\alpha t$ in degrees, for $P = 2.5$ and $\gamma = 90^\circ$ with $n = \sqrt{2}$. It is evident from Fig.38 that the point around which the resultant vector rotates has come outside the sausage shaped operating curve. Fig.39 shows what happens to the angle β when the pivotal point O Fig.41 comes inside the operating sausage. There is a permanent phase change of -2π radians. Incidentally, this condition has occurred for a change in γ of only 35' of arc from the 90° value chosen in Fig.38 to the 89° 25' value chosen in Fig.39. It must be remembered that a condition similar to that shown in Fig.39 can only be obtained when P is such that the maximum of $Pe^{-\alpha t} \sin n\alpha t > 1$, as the carrier amplitude has been taken as unity so that P is the relative noise to carrier ratio. Fig.40 shows the operating diagram which produced a permanent -2π radian phase change in the angle β shown in Fig.39.

We now ask ourselves what is the significant difference between the audio noise spectra produced by Figs. 38 and 39 respectively. First, we note that the time duration of the pulse of β in Figs. 38 and 39 is very small compared with the upper cut-off frequency of the audio circuits whether these consist of a low pass filter, a de-emphasis circuit or just a listener's ear. In this connection we find that for a $\pm$ 75 kc/s I.F. circuit with $n = \sqrt{2}$

$$n\alpha t = 75 . \pi \sqrt{2} . t \text{ kiloradians} \quad \text{.......} (4)$$

Now in both Figs. 38 and 39 β has gone through its cycle of change and returned virtually to a state of rest when $n\alpha t = 90^{\circ} = \frac{\pi}{2}$, whence $t = 4.7$ μS.

If the audio cut-off were as high as 15 kc/s this periodicity would amount to 66.7 μS which shows how relatively very short is the duration of the interfering noise. This means that virtually Fig.38 may be regarded as a unit impulse whilst Fig.39 may be taken as a unit step.

It is well known that the spectrum of unit impulse is uniform whilst that of unit step is a hyperbola. Now the discriminator output voltage is from appendix 3, equation 36

$$V_2(t) = \frac{A}{2\pi} \cdot \frac{d\beta}{dt} \quad \text{.......} (5)$$

so that the spectrum of the output voltage from Fig.38 will, by the rules of Heaviside analysis, be proportional to p as the operation of differentiation is equivalent to a multiplication by the differential operator $p = \frac{d}{dt}$. Thus Fig.38 gives the well known triangular spectrum wherein the height of the spectral lines is proportional to $p = j\omega$. Now in the case of Fig.39 with a hyperbolic spectrum proportional to $\frac{1}{p}$, the spectrum of the output voltage will be uniform, being obtained by the product of $\frac{1}{p}$ and p. Thus we see that when the peak value of the noise is greater than that of the carrier; and when the random phase angle, γ, has a suitable value such that the point on the end of the carrier vector about which pivots the resultant of carrier and noise vectors comes within the sausage shaped locus of the head of the noise vector, the spectrum of the audio output voltage changes quite suddenly from a triangular form to a uniform shape. It will be evident that the pops of interference are due to uniform spectra deriving from cases similar to that dealt with in Figs. 39 and 40 whilst clicks are due to triangular spectra deriving from low noise to signal ratios or from cases similar to that shown in Fig. 38.

3. The theoretical Instantaneous Frequency Deviation can be very great.

It may also be seen from a cursory inspection of Figs. 38 and 39 that the instantaneous frequency deviation can quite easily exceed the discriminator width but of course the output voltage cannot exceed that corresponding with this same width. For instance, the maximum theoretical frequency deviation in Fig. 38 is about 900 kc/s whilst in Fig. 39 it is about 1900 kc/s.

4. The Improvement Threshold

We have explained the occurrence of pops as distinct from clicks in the audio output from a F.M. receiver. These pops, containing as they do, far stronger low frequency components than do clicks, sound very much more annoying - in fact they are the same as the pops of interference in an A.M. system. Thus the F.M./A.M. improvement threshold occurs at that input carrier to noise ratio at which pops emerge from the output. This condition is attained when, as previously stated, the maximum of $P\,e^{-\alpha t} \sin n\alpha t > 1$. For $n = \sqrt{2}$ this gives $P > 2.4$, and for $Q = 50$ we get from equation 3 an input carrier to noise ratio of $\eta < 0.0177$. It is easily seen that for constant radio interference input amplitude, the carrier strength required to exceed the improvement threshold must be greater, the greater the deviation employed, as this will require a greater bandwidth and thus will result in a greater I.F. output noise peak, all other things remaining constant. Of course the modulation output will be higher but the advent of pops will occur more frequently unless the carrier strength is raised to that required by the new value of improvement threshold.

5. Rate of increase of Pops with I.F. output noise.

It is of interest to know how the percentage of pops per total number of interfering clicks and pops rises with increasing values of P, or in other words, according to what law does F.M. degenerate when the improvement threshold is not maintained. Fig. 41 shows the operating angle of the noise vector from an arbitary initial zero axis taken as colinear with the carrier vector. This operating angle is $\gamma - \pi - \psi$.

Now for a pop to occur it is evident that we must have

$$P\, e^{-\alpha t} \sin n\alpha t > 1 \quad \text{(that is, greater than carrier)}$$

when $\gamma - \pi - \psi = -\pi$ (that is, when noise vector is coincident with, but in opposition to the carrier vector)

We have then, from the second relationship,

$$\psi = \gamma \quad \text{.......... (6)}$$

$$\text{or } \arctan\left(\frac{2Q}{n} \tan n\alpha t\right) = \gamma \quad \text{.......... (7)}$$

whence

$$n\,\alpha t = \arctan\left(\frac{n}{2Q} \tan \gamma\right) \quad \text{.......... (8)}$$

If we call ρ this particular value of $n\alpha t$ we may write the inequality above as

$$P\, e^{-\frac{\rho}{n}} \sin \rho > 1$$

If we plot the curve P as a function of γ for

$$P\, e^{-\frac{\rho}{n}} \sin \rho = 1 \quad \text{.......... (9)}$$

we can find the arc containing within it those values of the random phase angle γ which for each value of P will produce pops - all the arcs left within the 360° will produce clicks. This curve is shown in Fig.42. Fig.43 is derived directly from Fig.42 and shows that the percentage of pops rises linearly with P, the I.F. output noise to carrier ratio, except for a rapid increase where P is close to the initiating value of 2.4.

6. A receiver which produces no Pops without Frequency Modulation of the Carrier.

One further point should be mentioned. All the foregoing analysis has applied to a F.M. receiver containing bandpass coupled circuits in the I.F. stage. If this stage had contained only a single or a cascade of single tuned circuits all aligned on the centre frequency, no pops would occur because the phase angle of the output noise from such a circuit due to a transient input does not vary with time but is a constant. In this case pops would only occur during modulation and would thus be less annoying than in a receiver containing I.F. bandpass coupled circuits. This may well be a point worth noting when executing a F.M. receiver design, but of course, the selectivity obtainable with single tuned circuits is far less desirable than that to be achieved with coupled tuned circuits.

7. Experimental Confirmation

The foregoing discussion on pops and clicks has been qualitatively confirmed by observation of the change in discriminator output waveform when clicks or pops are heard. For low repetition rates simultaneous aural and visual perception can easily identify the pop waveform as a single unidirectional pulse whereas that of clicks is a bi-directional pulse which is the rate of change of the former.

Appendix 11

List of Symbols and Formulae

f_o	$= \frac{\omega_o}{2\pi}$ = intermediate frequency
Q	= quality factor of uncoupled I.F. tuned circuits
K	= coupling factor of I.F. coupled tunod circuits
n	= KQ, the coupling parameter
L	= the common value of the coupled inductances of the I.F. tuned coupled circuits
p	= the operator $\frac{d}{dt}$ (p becomes $j\omega$ for sinusoidal forces)
Δf	= I.F. half bandwidth for 3 db. reduction in response from that at f_o.
$F_1(n)$	= a function of the coupling parameter plotted in Fig.7
α	$= \frac{\omega_o}{2Q} = \frac{2\pi\Delta f}{F_1(n)}$
δ'	= arctan n
δ	$= \arctan \frac{n}{1 - x}$
f_r	= pulse repetition frequency of the impulsive interference
η	= ratio of amplitude of wanted carrier to unit step of interference
x	$= \frac{\omega_a}{\alpha} = \frac{f_a}{\Delta f} F_1(n)$
f_c	= cut-off frequency of a low pass filter used in conjunction with de-emphasis in the audio portion of a F.M. receiver
f_a	$= \frac{\omega_a}{2\pi}$ = audio or video low pass filter out-off frequency or reciprocal of the product of 2π and the de-emphasis time constant or $\frac{1}{2\pi RC}$ where R and C are the de-emphasis resistance and capacitance respectively
f'_a	= a modified value of f_a for converting low pass filter formulae to de-emphasis formulae or to aural results
ω	= any angular frequency
N	= a noise voltage integrated over a frequency spectrum
γ	= the phase angle between carrier and interference vectors

β = the phase modulation produced by a noise vector

A = the discriminator co-efficient in volts out per volt in per kilocycle per second frequency deviation

$\phi(j\omega)$ = the frequency mate of a voltage time function, $V(t)$

Z = transfer impedance of a band pass coupled circuit network

S = the slope in amperes per volt of an ideal pentode valve

u = a variable of integration

y = $n\alpha t$ radians

θ = random phase angle between a wanted carrier and the oscillation of a noise pulse in an electric circuit

θ' = $\pi - \arccos \dfrac{1}{2C}$

C = a wanted carrier amplitude

R = the resultant vector of the wanted carrier and the noise pulse envelope vector

$\overline{R}$ = the mean value of R over the range of all possible values of θ

E = an elliptic integral

M_p and M_n = the mean of all positive and negative going pulses, respectively

e = an I.F. output voltage

e_1 = the A.M. I.F. wanted signal output voltage amplitude

e_2 = the F.M. discriminator wanted signal output voltage amplitude

$e(t)$ = the A.M. I.F. noise output pulse

$V_1(t)$ = the A.M. envelope of $e(t)$ after detection

$V_2(t)$ = the F.M. discriminator output noise pulse envelope

$\overline{e_1^2}$ = the A.M. mean square I.F. output signal voltage

$\overline{e_2^2}$ = the F.M. mean square discriminator output signal voltage

$V_{1a}(t)$ = the A.M. audio noise voltage as a time function

$V_{2a}(t)$ = the F.M. audio noise voltage as a time function

$\overline{V_{1a}^2}$ = the A.M. mean square audio noise voltage

$\overline{V_{2a}^2}$ = the F.M. mean square audio noise voltage

$\overline{V_1^2}$ = the A.M. mean square I.F. detected noise voltage

$\overline{V_2^2}$ = the F.M. mean square discriminator output noise voltage

U = the percussion or area of a unit impulse

B = the maximum height of an impulse of percussion U and duration τ

τ = the time duration of a unit impulse

T_d = a detector discharge time constant

T_c = a detector charge time constant

W = a pulse width

J = a mean detected current

$\mu = \frac{W}{T_c}$

$\sigma = (1/f_r - W)/T_d$

(t) represents a time function, usually due to unit step exciting force

[p] represents the operational equivalent to a time function, thus

$$g(t) \doteq g[p]$$

where $\doteq$ means "operationally equivalent to"

*(t) represents the time function response to unit impulse

P = coefficient of the IF noise output representing the noise to carrier ratio

ψ = the variable phase angle of the IF noise output with respect to IF centre frequency

Where to find it	Quantity Expressed	Mathematical expression for it	Remarks
Appendix 3, Equation 8	e(t), the I.F. output voltage due to unit step input current.	$\omega_o L e^{-\alpha t} \sin n\alpha t \cos \omega_o t$	
Appendix 3, Equation 9	$V_1(t)$, the envelope of e(t)	$\omega_o L e^{-\alpha t} \sin n\alpha t$	After A.M. detection in company with a steady carrier wave of amplitude greater than its own peak value.
Appendix 3, Equation 37	$V_2(t)$, the F.M. discriminator noise output voltage	$\frac{A\alpha(1+n^2)^{\frac{3}{2}}}{2\pi\eta nQ} e^{-\alpha t} \sin(n\alpha t - \delta')$	Wanted carrier amplitude assumed greater than the peak value of $V_2(t)$
Appendix 3, Equation 10	Peak V_1, the greatest value of $V_1(t)$	$\frac{\omega_o L n}{e^{\delta'/n}\sqrt{1+n^2}}$	
Appendix 3, Equation 38	Peak V_2, the greatest value of $V_2(t)$	$\frac{A\alpha^2(1+n^2)}{2\pi^2\eta f_o}$	
Appendix 3, Equation 11	I.F. Output Peak $\left(\frac{\text{Signal}}{\text{Noise}}\right)_{\text{A.M.}}$	$\eta \frac{f_o F_1(n) e^{\frac{\delta'}{n}}}{2\Delta f\sqrt{1+n^2}}$	This is for 100% modulation
Appendix 3, Equation 40	I.F. Output Peak $\left(\frac{\text{Signal}}{\text{Noise}}\right)_{\text{F.M.}}$	$\eta \frac{f_o F_1^2(n)}{2(1+n^2)\Delta f}$	This is for 100% modulation, that is, for a frequency deviation of Δf

Where to find it	Quantity Expressed	Mathematical expression for it	Remarks
Appendix 3, Equation 16	I.F. Output RMS $\left(\frac{\text{Signal}}{\text{Noise}}\right)$ A.M.	$\eta \frac{f_o}{\sqrt{f_r \Delta f}} \sqrt{\frac{\pi F_1(n)}{1+n^2}}$	This is for 100% modulation. This ratio should be increased by 4 db. to convert from peak to mean RMS noise due to random phasing of the individual input noise pulses.
Appendix 3, Equation 44	I.F. Output RMS $\left(\frac{\text{Signal}}{\text{Noise}}\right)$ F.M.	$\eta \frac{f_o}{1+n^2} \sqrt{\frac{\pi F_1^3(n)}{f_r \Delta f}}$	This is for 100% modulation. This ratio should be increased by 4 db. to convert from peak to mean RMS noise due to random phasing of the individual input noise pulses.
Appendix 3, Equation 17	I.F. Output Noise $\left(\frac{\text{Peak}}{\text{RMS}}\right)$ A.M.	$2e^{-\delta'/n} \sqrt{\frac{2\pi\Delta f}{f_r F_1(n)}}$	Add 4 db. for peak to mean RMS noise.
Appendix 3, Equation 45	I.F. Output Noise $\left(\frac{\text{Peak}}{\text{RMS}}\right)$ F.M.	$2\sqrt{\frac{2\pi\Delta f}{f_r F_1(n)}}$	Add 4 db. for peak to mean RMS noise.
Appendix 2, Equation 1	A.F. Output Peak $\left(\frac{\text{Signal}}{\text{Noise}}\right)$ A.M.	$\eta \frac{n^2 f_o}{2f_a(1+n^2)\sin\delta} \cdot \frac{1}{\sin\delta\, e^{-\omega_a t} - e^{-\alpha t}\sin(n\alpha t+\delta)}$	For 100% modulation and wherein t has that value which satisfies the transcendental equation $\frac{x\sin\delta}{\sqrt{1+n^2}} e^{(\alpha-\omega_a)t} = \sin(n\alpha t-\delta-\delta')$, $0<t<\frac{\pi}{n\alpha}$

Where to find it	Quantity Expressed	Mathematical expression for it	Remarks
Appendix 2, Equation 2.	A.F. Output Peak $\left(\frac{\text{Signal}}{\text{Noise}}\right)_{F.M.}$	$\eta \dfrac{n^2 f_o F_1(n)}{2f_a(1+n^2)^{3/2} \sin\delta} \cdot \dfrac{1}{\sin(\delta-\delta')e^{-\omega_a t} - e^{-\alpha t}\sin(n\alpha t+\delta-\delta')}$	For 100% modulation, that is, for a frequency deviation of Δf and wherein t has that value which satisfies the transcendental equation $\dfrac{x\sin(\delta-\delta')}{\sqrt{1+n^2}} e^{(\alpha-\omega_a)t} = \sin(n\alpha t+\delta-2\delta')$, $o < t < \frac{\pi}{n\alpha}$
Appendix 3, Equation 26.	A.F. Output RMS $\left(\frac{\text{Signal}}{\text{Noise}}\right)_{A.M.}$	$\eta\pi f_o \sqrt{\dfrac{2nx}{(1+n^2)f_r f_a}} \cdot \dfrac{1}{\sqrt{2n \arctan \dfrac{2x}{(1+n^2)-x^2} + 2\cdot 3 \log_{10} \dfrac{x^2+2nx+1+n^2}{x^2-2nx+1+n^2}}}$	100% modulation is assumed. Increase this ratio by 4 db. to convert from peak to mean RMS noise due to random phasing of noise input pulses. When using an audio (or video) low pass filter of cut-off f_c let $f_a = f_c$. When using de-emphasis of time constant RC let $f_a = \frac{1}{4RC}$. To obtain the aural signal to noise ratio consider the ear as a 5 kc/s low pass filter. Add the 4 db. If de-emphasis is used let $f_a = \frac{1}{4RC} \arctan 20\,RC$ where RC is in milliseconds, Δf, f_o and f_r in kilocycles per second. Add the 4 db.
Appendix 3, Equation 27.	A.F. Output RMS $\left(\frac{\text{Signal}}{\text{Noise}}\right)_{A.M.}$ for $x < 1$	$\eta \frac{\pi}{2} \cdot \sqrt{\dfrac{f_o}{f_r \cdot f_a}}$	This is the limiting case of the above formula when x is very small, but it applies with adequate accuracy if $x < 1$.

Where to find it	Quantity Expressed	Mathematical expression for it	Remarks
Appendix 3, Equation 53	A.F. Output RMS $\left(\frac{\text{Signal}}{\text{Noise}}\right)_{F.M.}$	$\eta\pi \frac{f_o F_1(n)}{1+n^2} \sqrt{\frac{2nx}{f_r \cdot f_a}} \cdot \frac{1}{\sqrt{2n \arctan \frac{2x}{(1+n^2)-x^2} - 2\cdot 3 \log_{10} \frac{x^2+2nx+1+n^2}{x^2-2nx+1+n^2}}}$	This is for 100% modulation, that is, for a frequency deviation of Δf. This ratio should be increased by 4 db. to convert from peak to mean RMS noise due to random phasing of the individual input noise pulses. When using an audio low pass filter of cut-off f_c let $f_a = f_c$. When using de-emphasis of time constant RC in conjunction with a low pass filter of cut-off f_c let $f_a = \frac{1}{2\pi RC}$ $\left[3(2\pi RCf_c - \arctan 2\pi RCf_c)\right]^{\frac{1}{3}}$ To obtain the aural signal to noise ratio consider the ear as an 8 kc/s low pass filter. Add the 4 db. If de-emphasis of time constant RC is used then let $f_a = \frac{1}{2\pi RC}\left[3(16\pi RC - \arctan 16\pi RC)\right]^{\frac{1}{3}}$ RC in mS., Δf, f_o, f_r in kc/s. Add the 4 db.
Appendix 3, Equation 54	A.F. Output RMS $\left(\frac{\text{Signal}}{\text{Noise}}\right)_{F.M.}$	$\frac{\eta\pi\sqrt{3}}{2} \cdot \frac{f_o \cdot \Delta f}{\sqrt{f_r \cdot f_a^3}}$	This is the limiting case of the above formula when x is small, but it applies with adequate accuracy if $x < 1$.

Where to find it	Quantity Expressed	Mathematical expression for it	Remarks
Appendix 2, Equation 3	Signal/Noise $\left(\frac{F.M.}{A.M.}\right)$ RMS	$\sqrt{3} \cdot \frac{\Delta f}{f_a}$	This assumes 100% modulation and identical I.F. and A.F. circuits in the two cases. Also the A.F circuits are assumed to consist of a low pass filter of cut-off frequency $f_a = f_c$. The I.F. circuits have been taken as wider than the audio circuits, that is, the formulae used to obtain the given expression are those limiting cases obtained for $x < 1$.
Appendix 2, Equation 4	Signal/Noise $\left(\frac{F.M.}{A.M.}\right)$ RMS	$\frac{\sqrt{\frac{\pi}{2}}}{\sqrt{\frac{f_c}{f_a} - \arctan\frac{f_c}{f_a}}} \cdot \frac{\Delta f}{f_a}$	This assumes 100% modulation, identical I.F. circuits, the same de-emphasis of time constant $\frac{1}{2\pi f_a}$ in both audio circuits, and a low pass filter of cut-off frequency f_c, in addition, in the audio circuits of the F.M. receiver. If the F.M. receiver audio circuits contain no low pass filter then let $f_c = \Delta f$. Also it is assumed that the I.F. half width is greater than audio.
Appendix 2, Equation 5	Signal/Noise $\left(\frac{F.M.}{A.M.}\right)$ RMS (Aural impression)	$\frac{\Delta f}{f_a}\sqrt{\frac{\arctan 5/f_a}{8/f_a - \arctan(8/f_a)}}$	This assumes 100% modulation, identical I.F. and A.F. circuits, the latter consisting of a de-emphasis circuit of time constant $\frac{1}{2\pi f_a}$. Also it is assumed that the I.F. half width is greater than audio.
Appendix 2, Equation 6	Signal/Noise $\left(\frac{F.M.}{A.M.}\right)$ RMS (Aural impression)	$\frac{\sqrt{5}\;\Delta f}{\sqrt{f_a^{3}\left(\frac{8}{f_a} - \arctan\frac{8}{f_a}\right)}}$	This assumes 100% modulation. A.M. A.F. circuits assumed limited only by the ear (5 kc/s). F.M. A.F. circuits with de-emphasis of time constant $\frac{1}{2\pi f_a}$ followed by the ear. Also the I.F. half width is assumed greater than audio.

Where to find it	Quantity Expressed	Mathematical expression for it	Remarks
Appendix 5, Equation 13	Envelope of $e_2(t)$	$S\frac{\omega_o^3 L^2}{8n\alpha}e^{-\alpha t}\sqrt{(n\alpha t)^2+4\sin^2 n\alpha t-2n\alpha t\sin 2n\alpha t}$	This is the envelope of the A.M. pulse due to unit step input, emerging from the output of two bandpass coupled circuit networks connected by an ideal pentode of mutual conductance S.
Appendix 5, Equation 19	Envelope of $e_2(t)$	$\frac{\omega_o L(1+n^2)}{4n^2}e^{-\alpha t}\sqrt{(n\alpha t)^2+4\sin^2 n\alpha t-2n\alpha t\sin 2n\alpha t}$	This is the envelope of the A.M. pulse due to unit step input, emerging from the output of two bandpass coupled circuit networks connected by an ideal pentode of such mutual conductance as to equate the steady state midband gain to that of a single bandpass coupled circuit network alone. This equation may be compared, therefore, with Appendix 3, equation 9.
Appendix 6, Equation 3	The mean value $\bar{R}$ of all the resultants R of a carrier vector and a noise vector having the same frequency but random phase angles.	$\frac{2}{\pi}(1+C)\,E\left(\frac{\pi}{2},\ \frac{2\sqrt{C}}{1+C}\right)$	$C \gg 1$.
Appendix 6, Equations 9 and 8	The means of the detected random amplitude noise pulses for positive and negative directions, respectively.	$M_p = \frac{2(1+C)}{\pi}E\left(\frac{\theta'}{2},\ \frac{2\sqrt{C}}{1+C}\right) - \frac{\theta'}{\pi}C$ $M_n = C+M_p - \bar{R}$	$C > 1$.
Appendix 10, Equation 3 and Paragraph 5	FM/AM improvement threshold.	$\eta = \frac{\sqrt{1+n^2}}{Qe^{\delta/n}}$	This is the RF input carrier to noise ratio above which the full FM/AM improvement is attained.

Appendix 12

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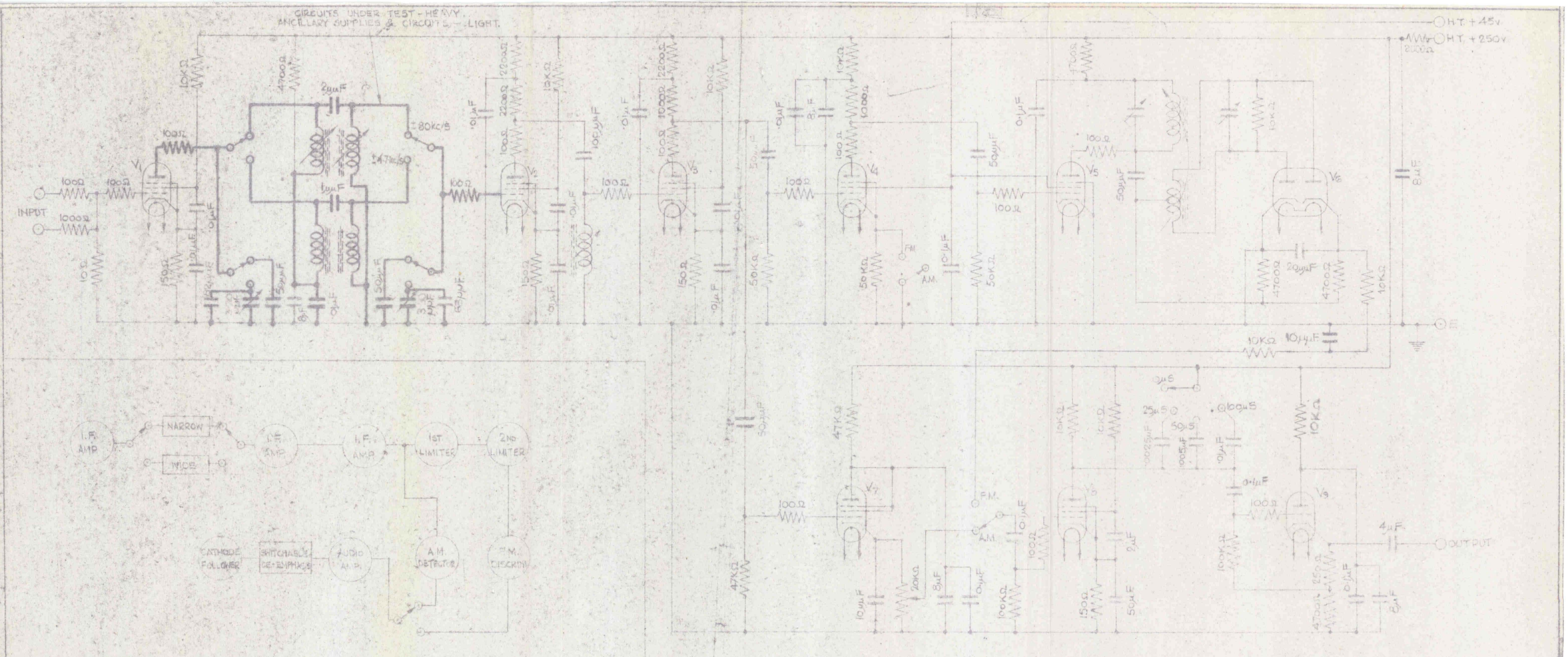

FIG. 1b BLOCK SCHEMATIC OF RECEIVER

FIG. 1a F.M./AM RECEIVER

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IMPULSIVE INTERFERENCE IN A.M AND F.M

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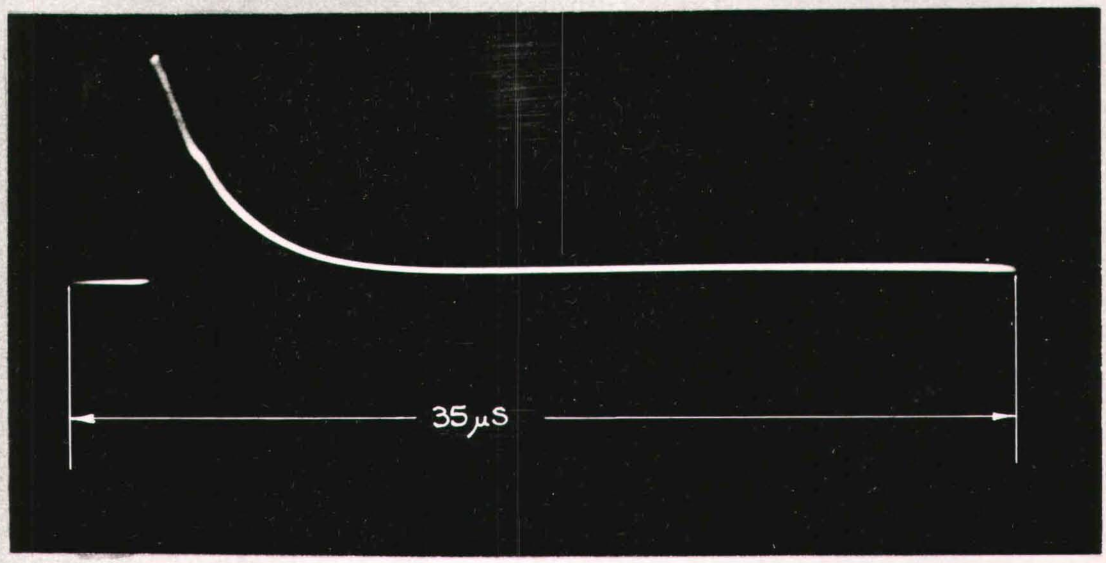

FIG. 2
WAVEFORM OF INPUT INTERFERENCE

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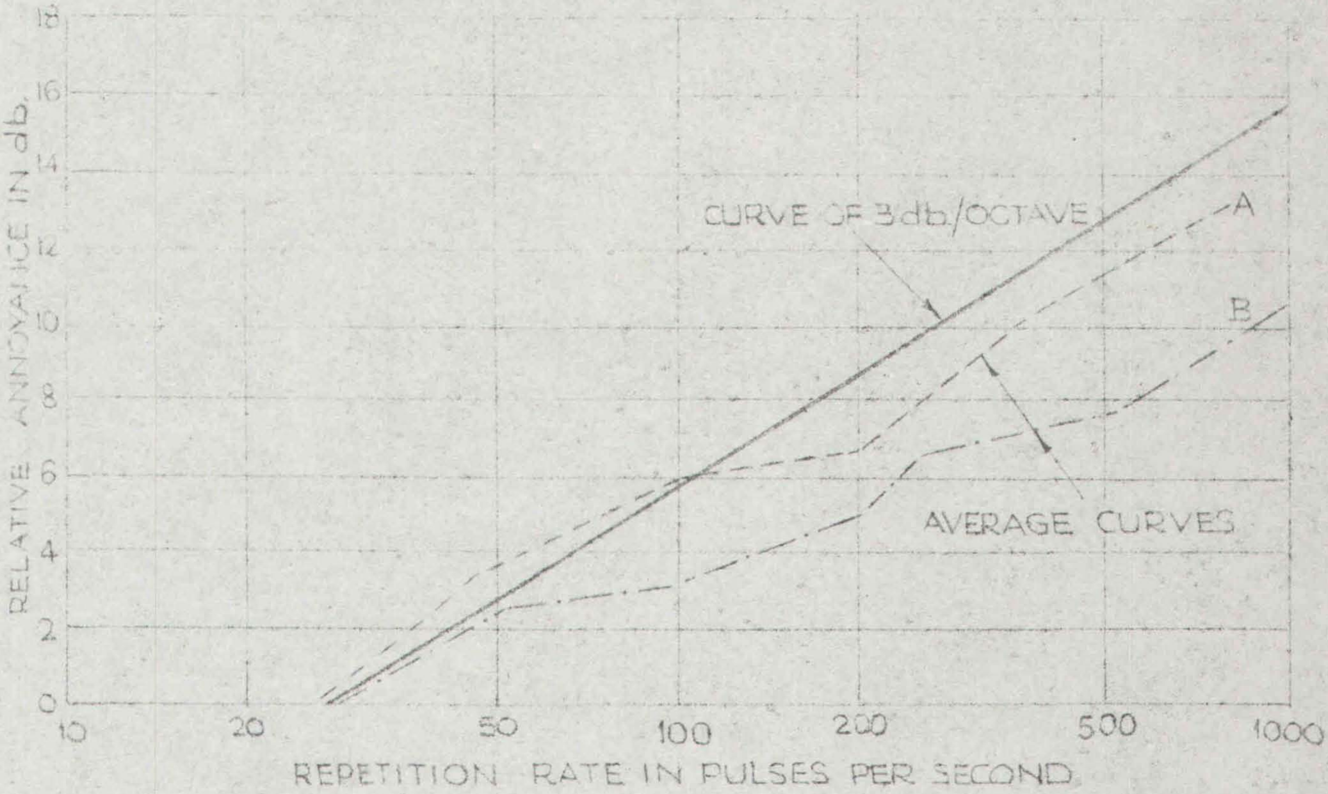

CURVE A – LEVEL OF EACH PRF COMPARED WITH 25 P.P.S. IN PRESENCE OF MUSICAL PROGRAMME.

CURVE B – LEVEL OF EACH PRF ADJUSTED TO GIVE EFFECT OF 'JUST DISTURBING' IN PRESENCE OF MUSICAL PROGRAMME.

FIG.3.

RELATIVE ANNOYANCE OF IMPULSIVE INTERFERENCE OF RANDOM AMPLITUDE WITH A MUSICAL PROGRAMME PRESENT.

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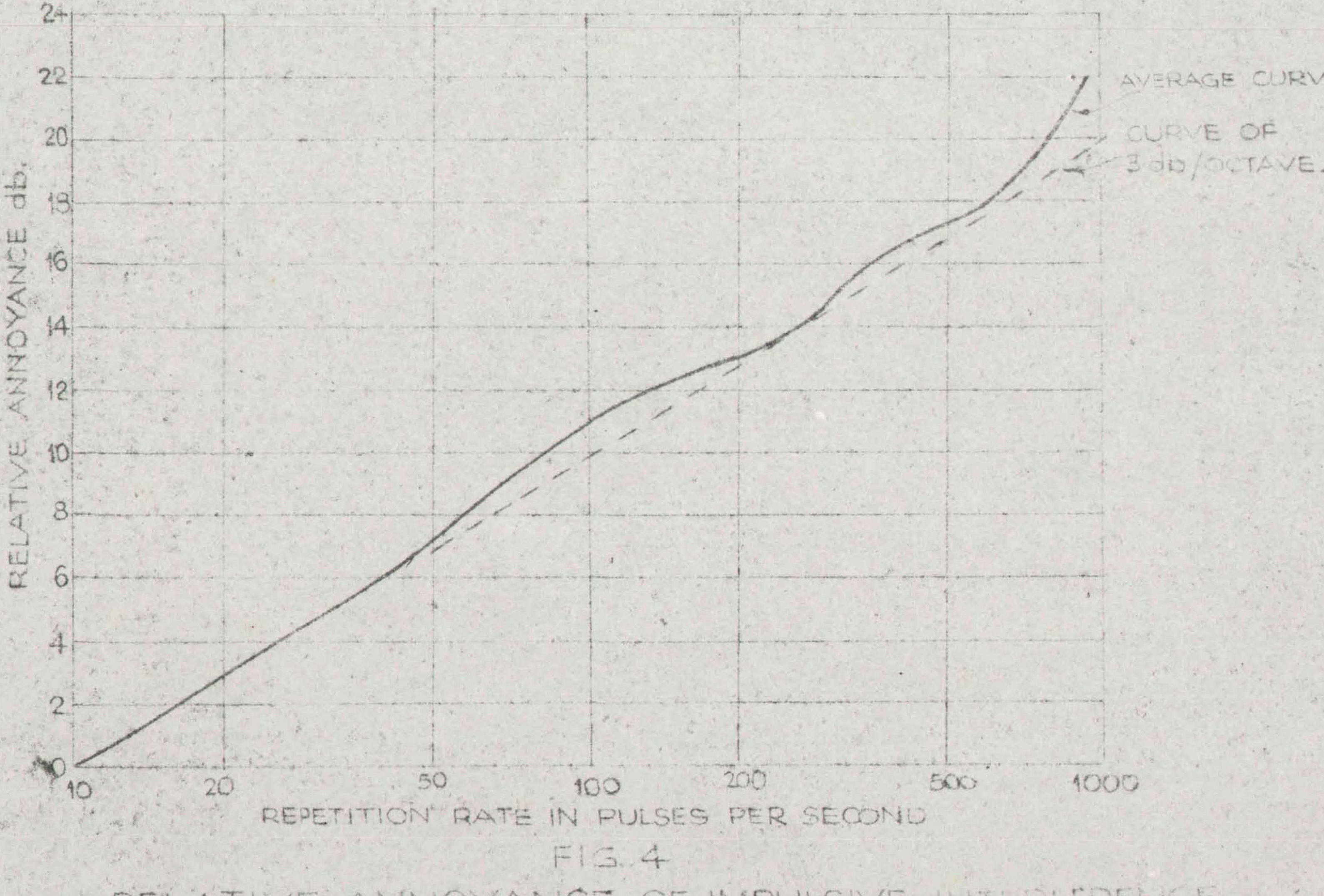

FIG. 4

RELATIVE ANNOYANCE OF IMPULSIVE INTERFERENCE OF UNIFORM HEIGHT WITH MUSICAL PROGRAMME PRESENT.

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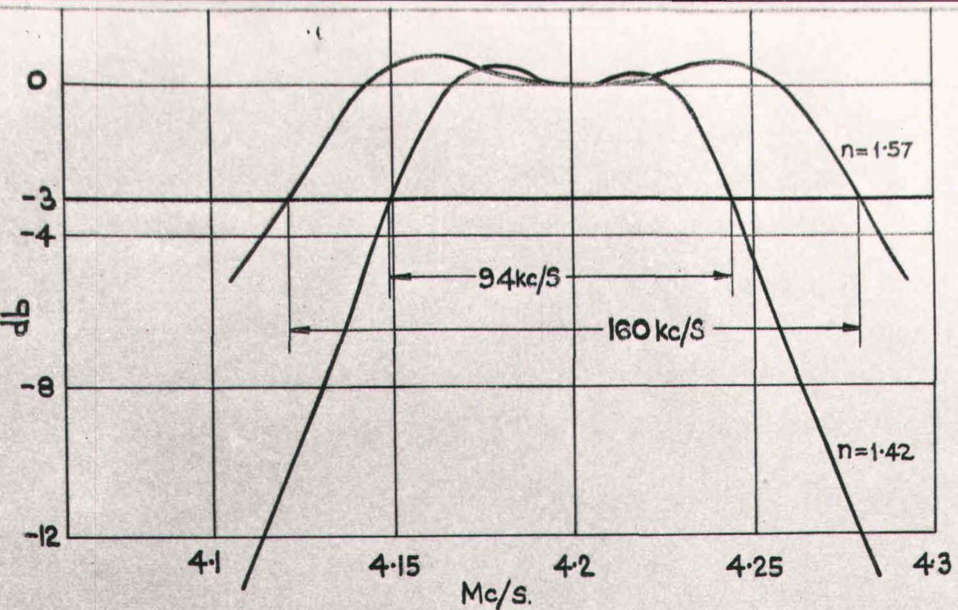
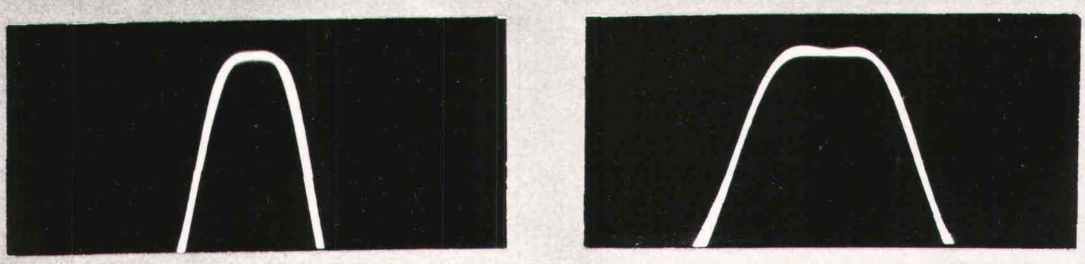
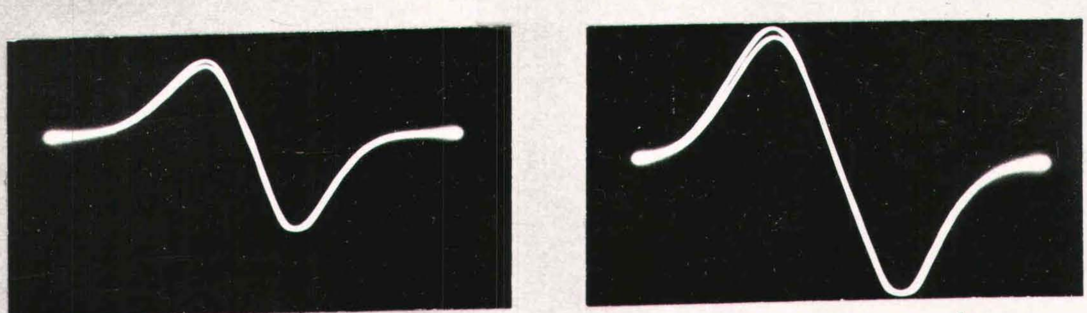


STATIC STEADY STATE RESPONSE OF I.F.
BANDPASS COUPLED CIRCUITS.



I.F. WIDTH 94 kc/s | I.F. WIDTH 160 kc/s
BANDPASS COUPLED CIRCUITS.



I.F. WIDTH 94 Kc/s | I.F. WIDTH 160kc/s
BANDPASS COUPLED CIRCUITS FOLLOWED BY DISCRIMINATOR.

DYNAMIC RESPONSES

FIG. 5.

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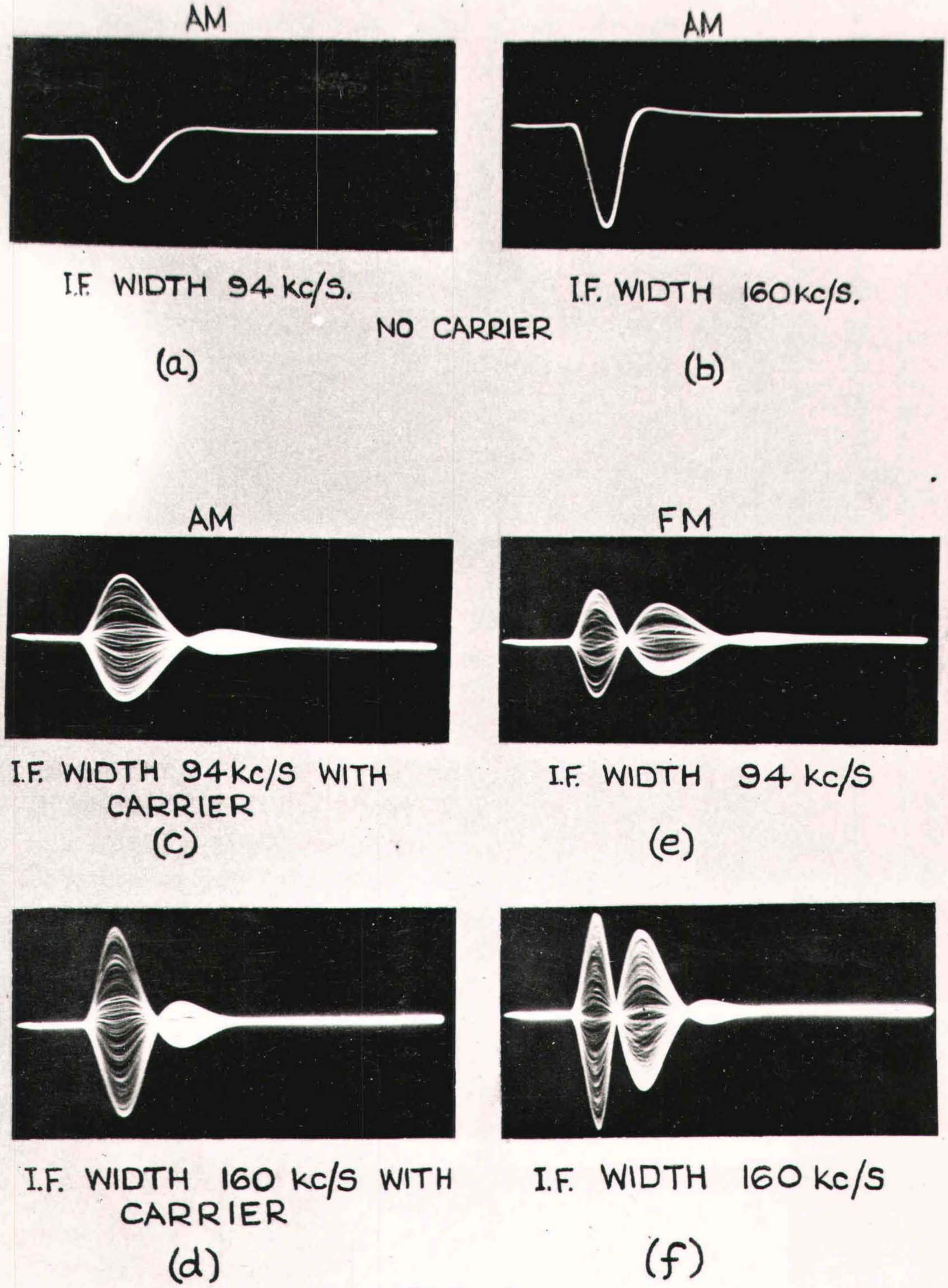

FIG. 6

DETECTED I.F. NOISE PULSES.

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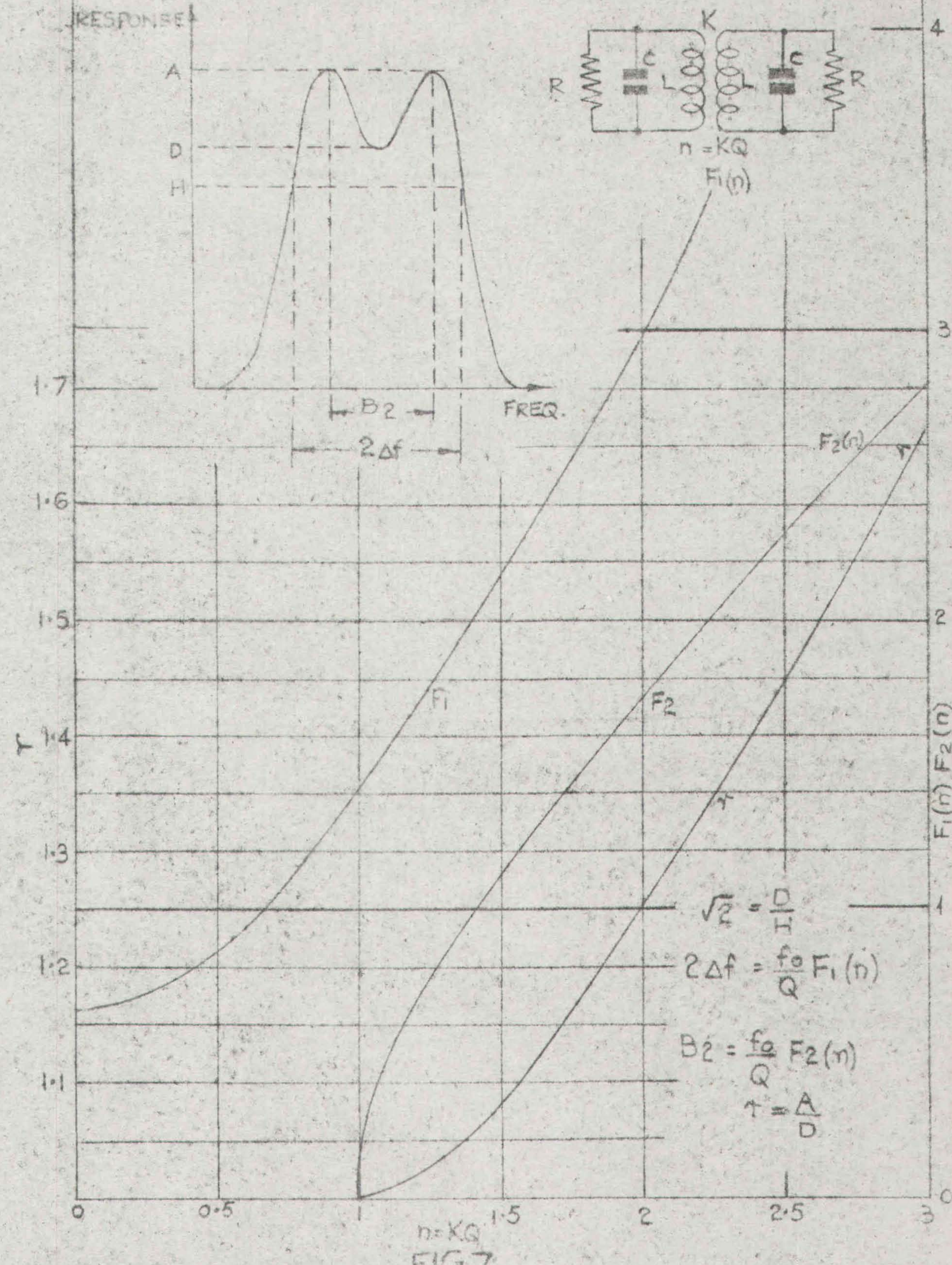

$\sqrt{2} = \frac{D}{H}$

$2\Delta f = \frac{f_0}{Q} F_1(n)$

$B_2 = \frac{f_0}{Q} F_2(n)$

$r = \frac{A}{D}$

FIG.7

COUPLED CIRCUIT DESIGN CHART.

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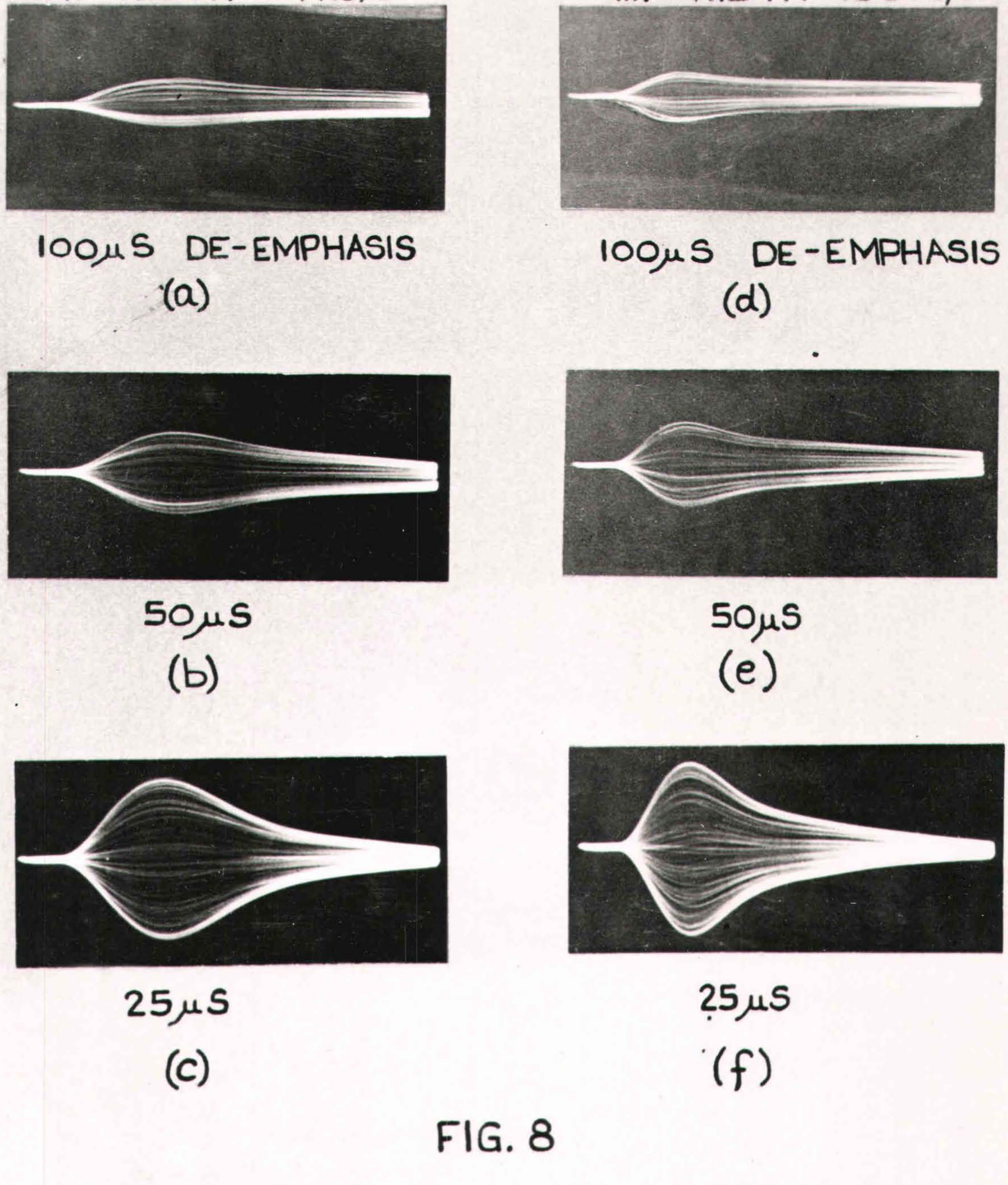

FIG. 8

A.F. OUTPUT NOISE PULSES
WITH CARRIER
A.M.

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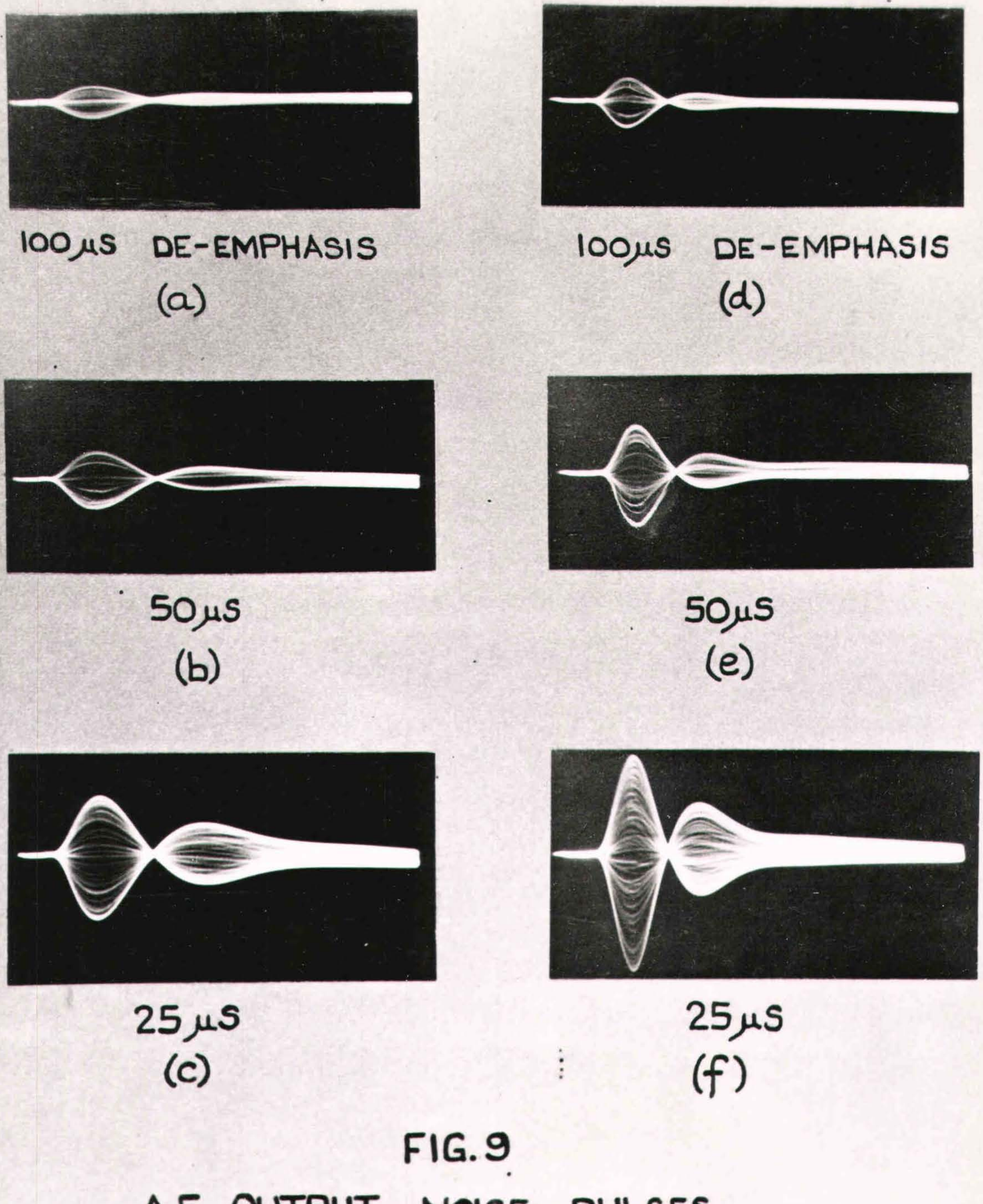

FIG. 9

A.F. OUTPUT NOISE PULSES
F.M.

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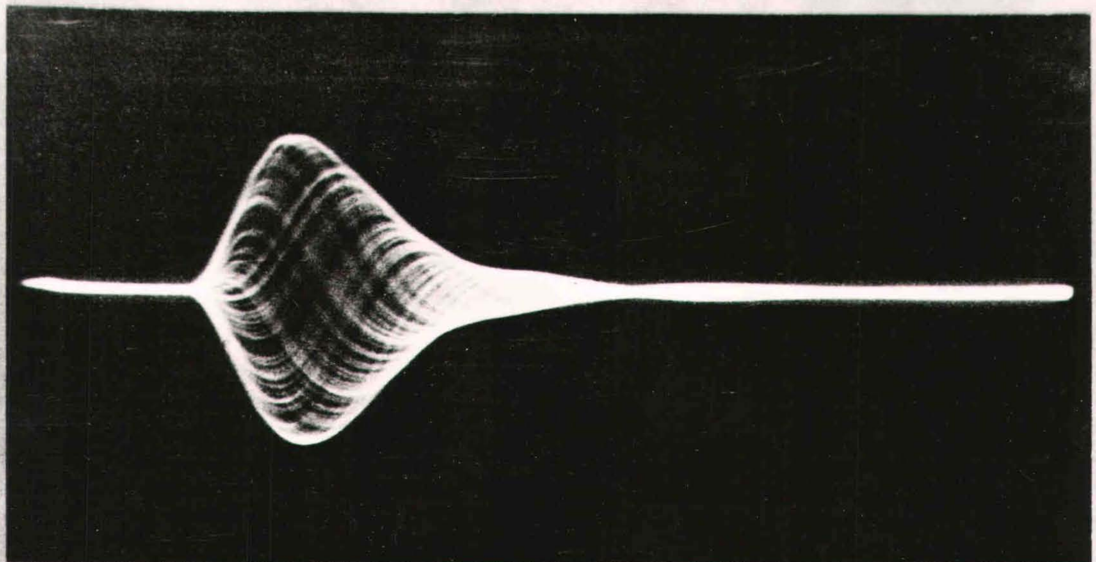
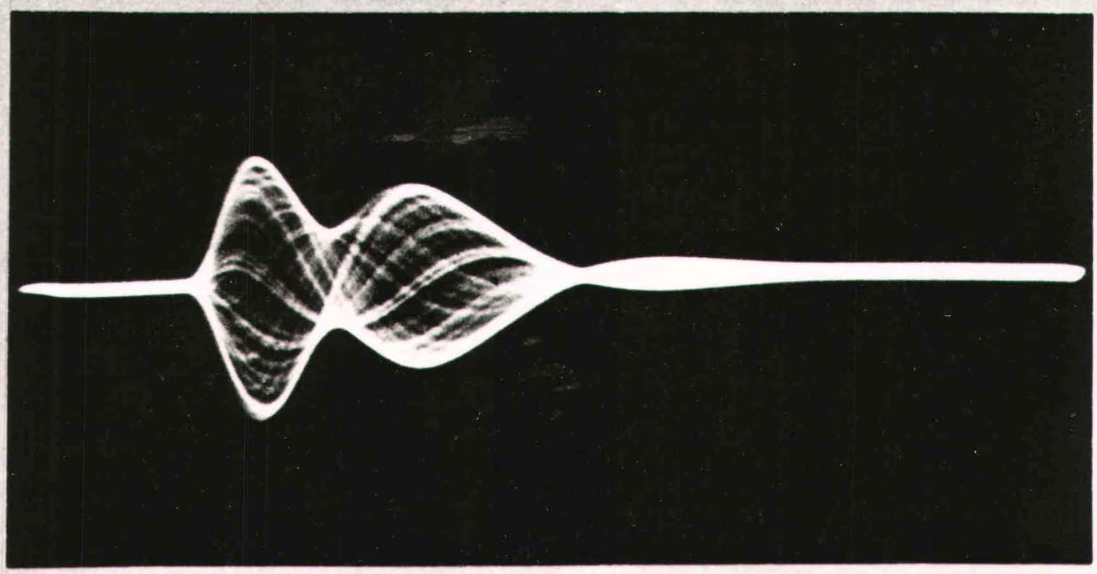


A.M. (a)



F.M. (b)

FIG. 10.
DETECTED I.F. NOISE PULSES IN PRESENCE OF CARRIER MIS-TUNED FROM I.F. CENTRE. I.F. WIDTH 94 Kc/S.

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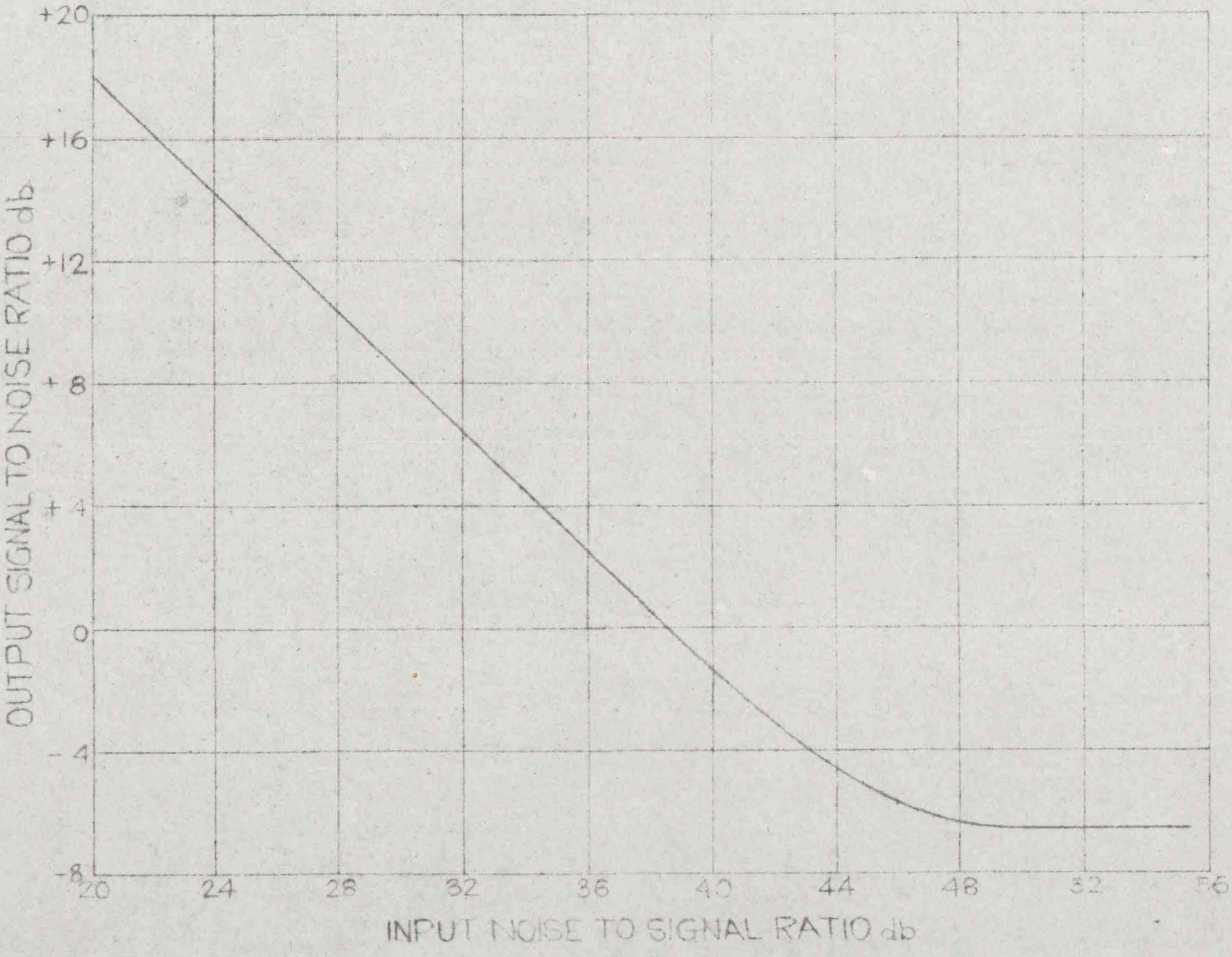

FIG.11

VARIATION OF F.M. I.F. OUTPUT PEAK SIGNAL TO NOISE RATIO WITH INPUT NOISE TO SIGNAL RATIO.

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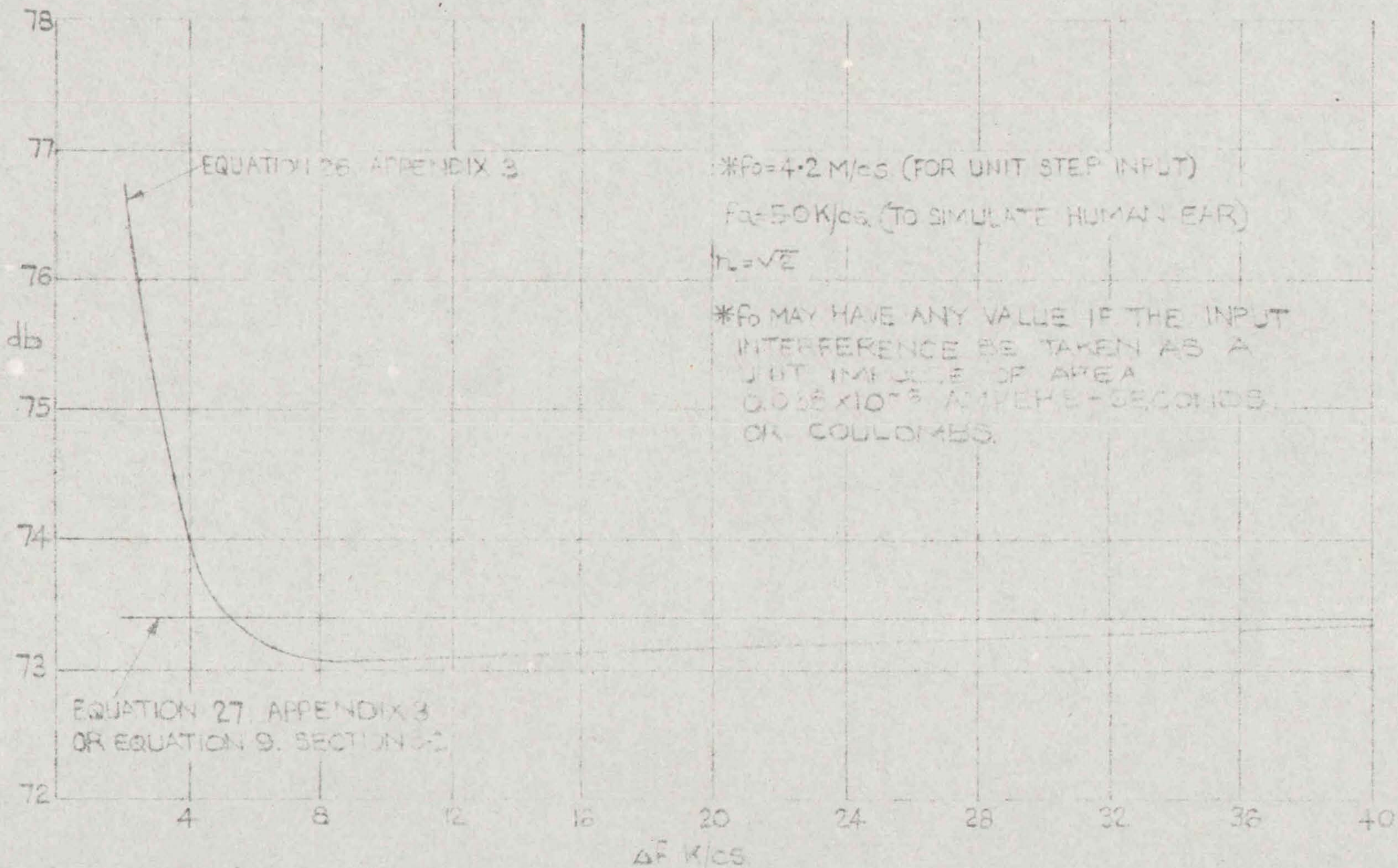

FIG.12

A.F. OUTPUT R.M.S. SIGNAL TO NOISE RATIO FOR A.M. RECEIVER

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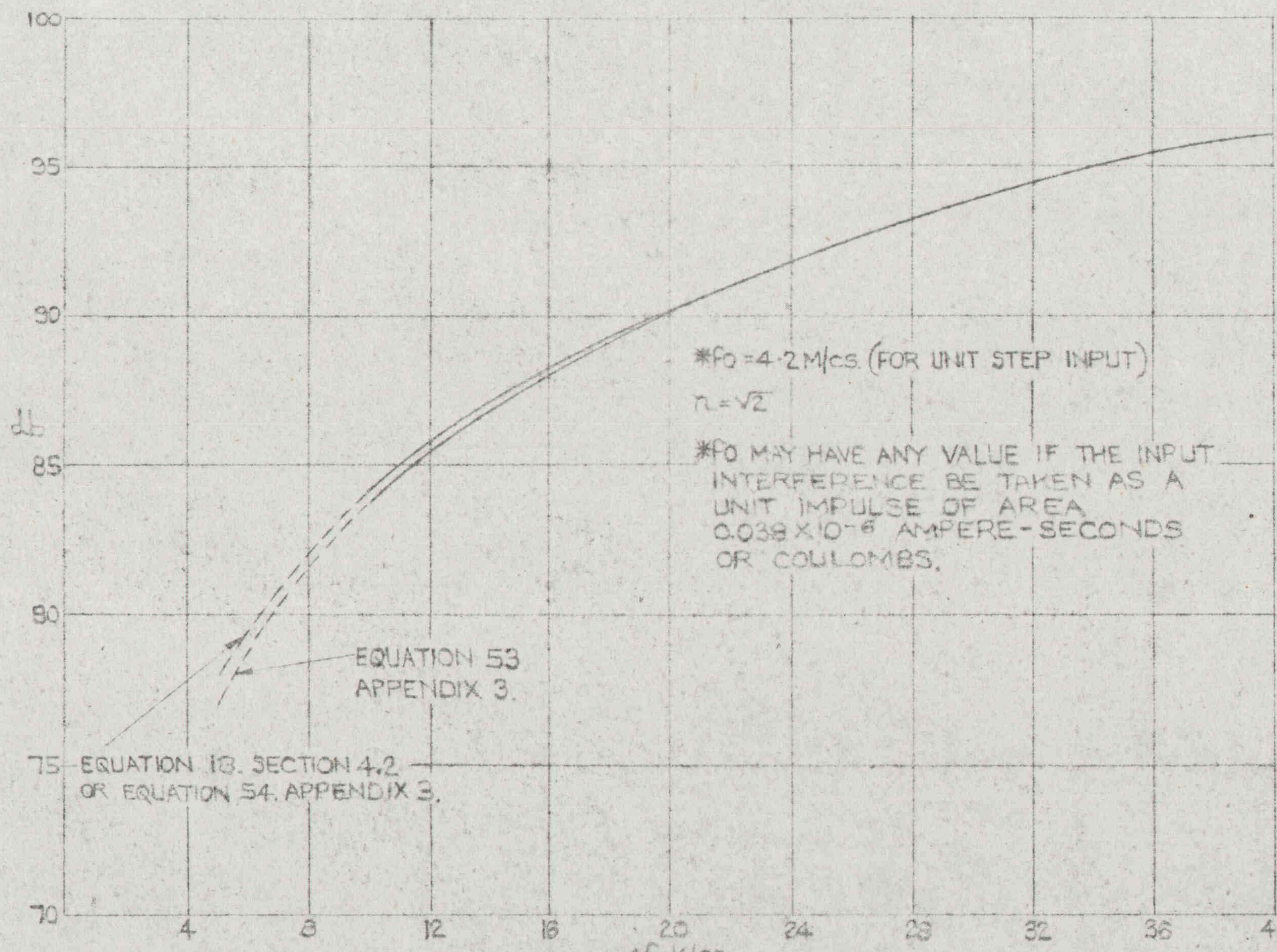

FIG. 13

A.F. OUTPUT R.M.S. SIGNAL TO NOISE RATIO FOR F.M. RECEIVER

WITH 50 μsec DE-EMPHASIS FOLLOWED BY 8K/C LOW PASS FILTER (TO SIMULATE HUMAN EAR)

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AP'D	

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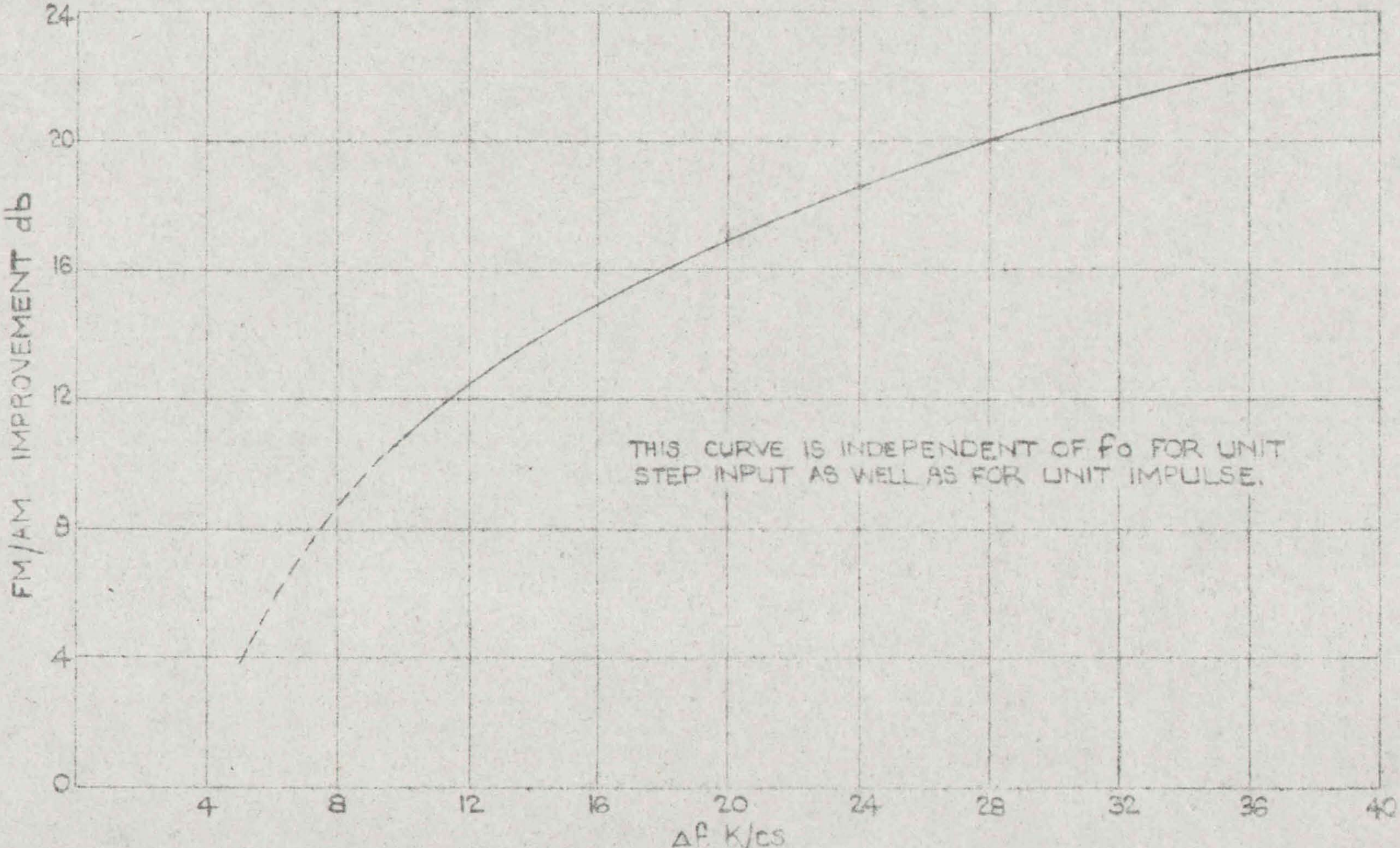

FIG. 14

R.M.S. $\frac{F.M.}{A.M.}$ SIGNAL TO NOISE RATIO IMPROVEMENT WITH VARYING I.F. BANDWIDTH
(DERIVED FROM FIGS. 12 & 13)

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CH'D	
AP'D	D.M.

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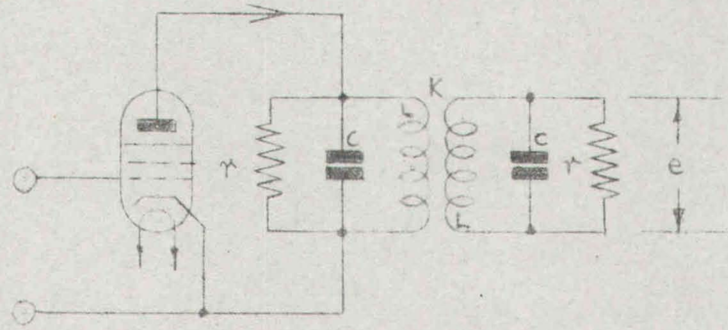

FIG. 15.

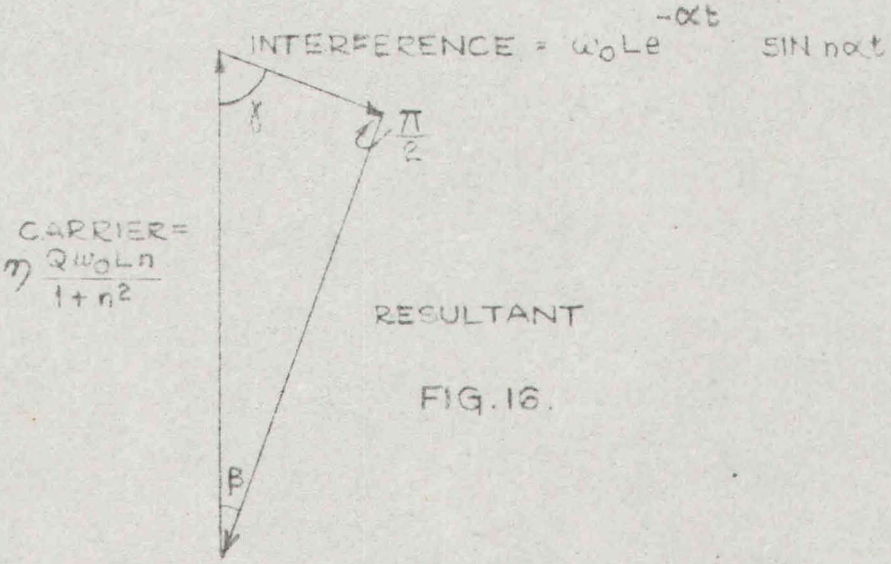

FIG. 16.

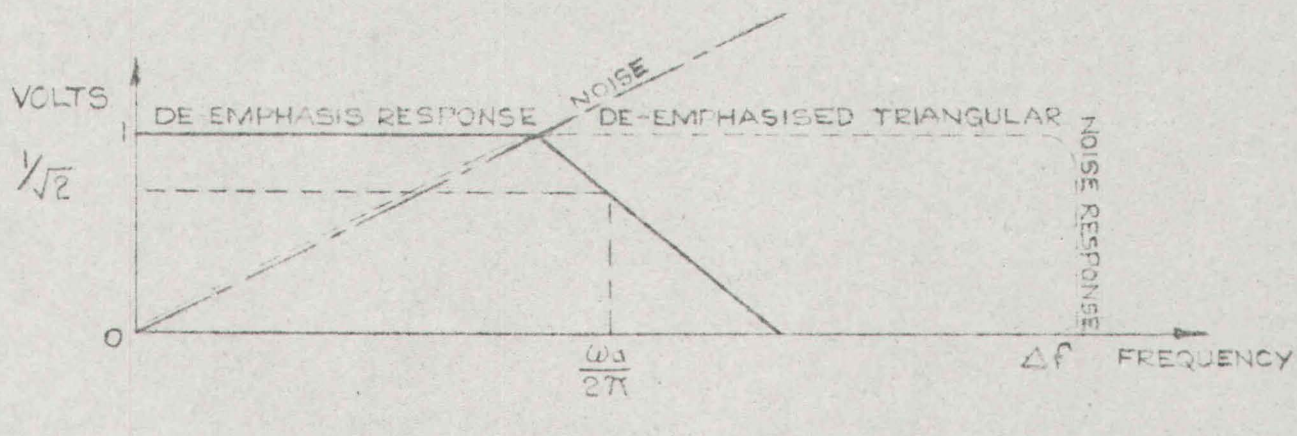

FIG. 17

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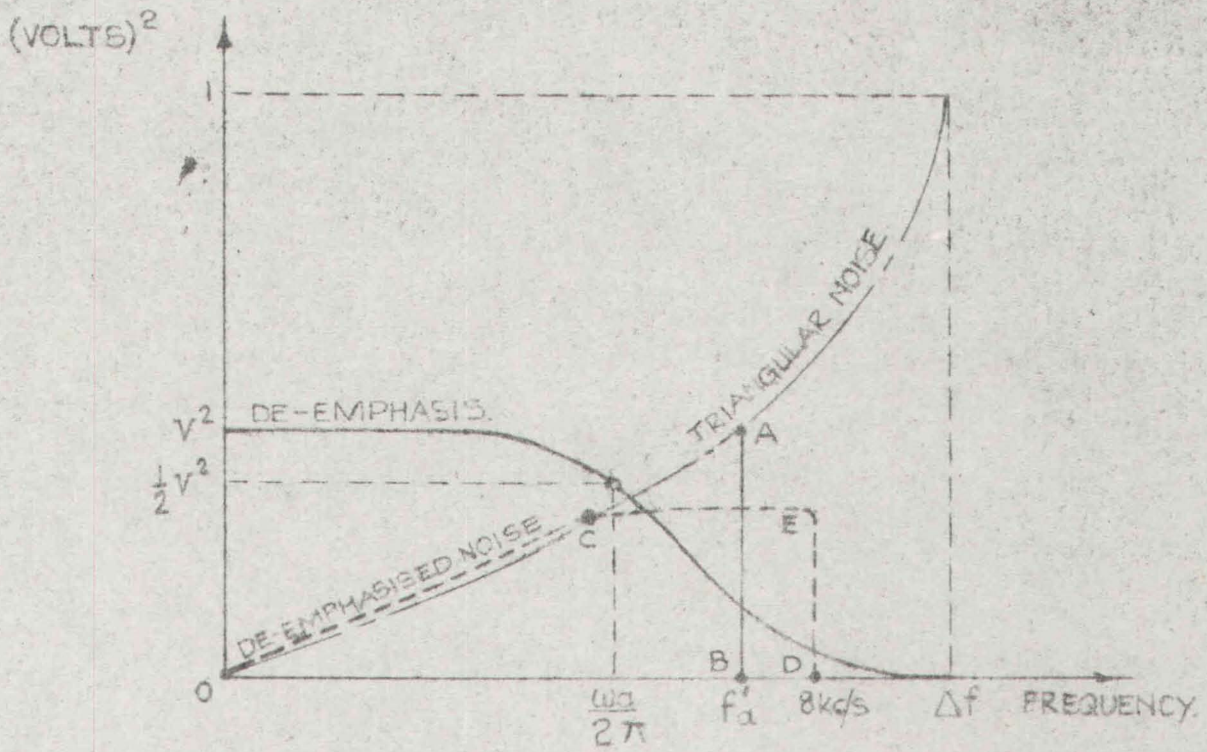

FIG. 18.

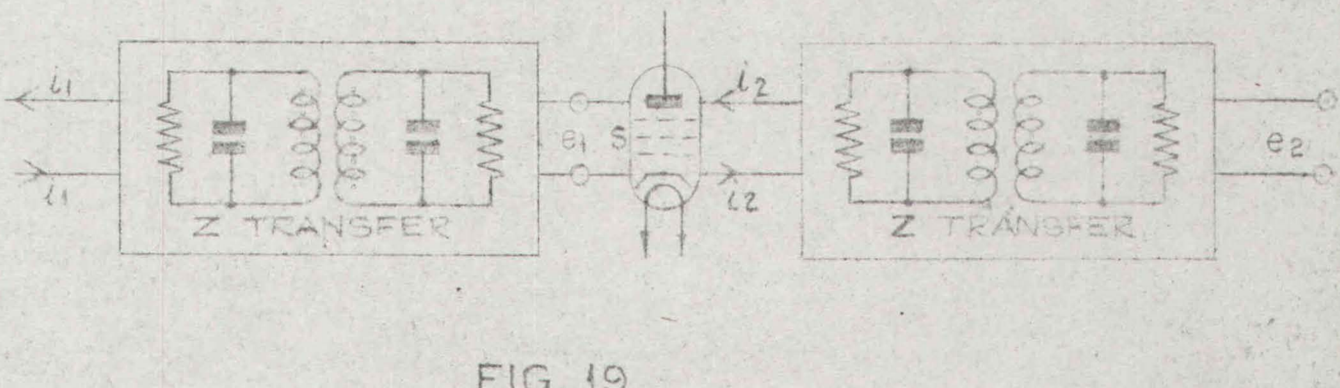

FIG. 19.

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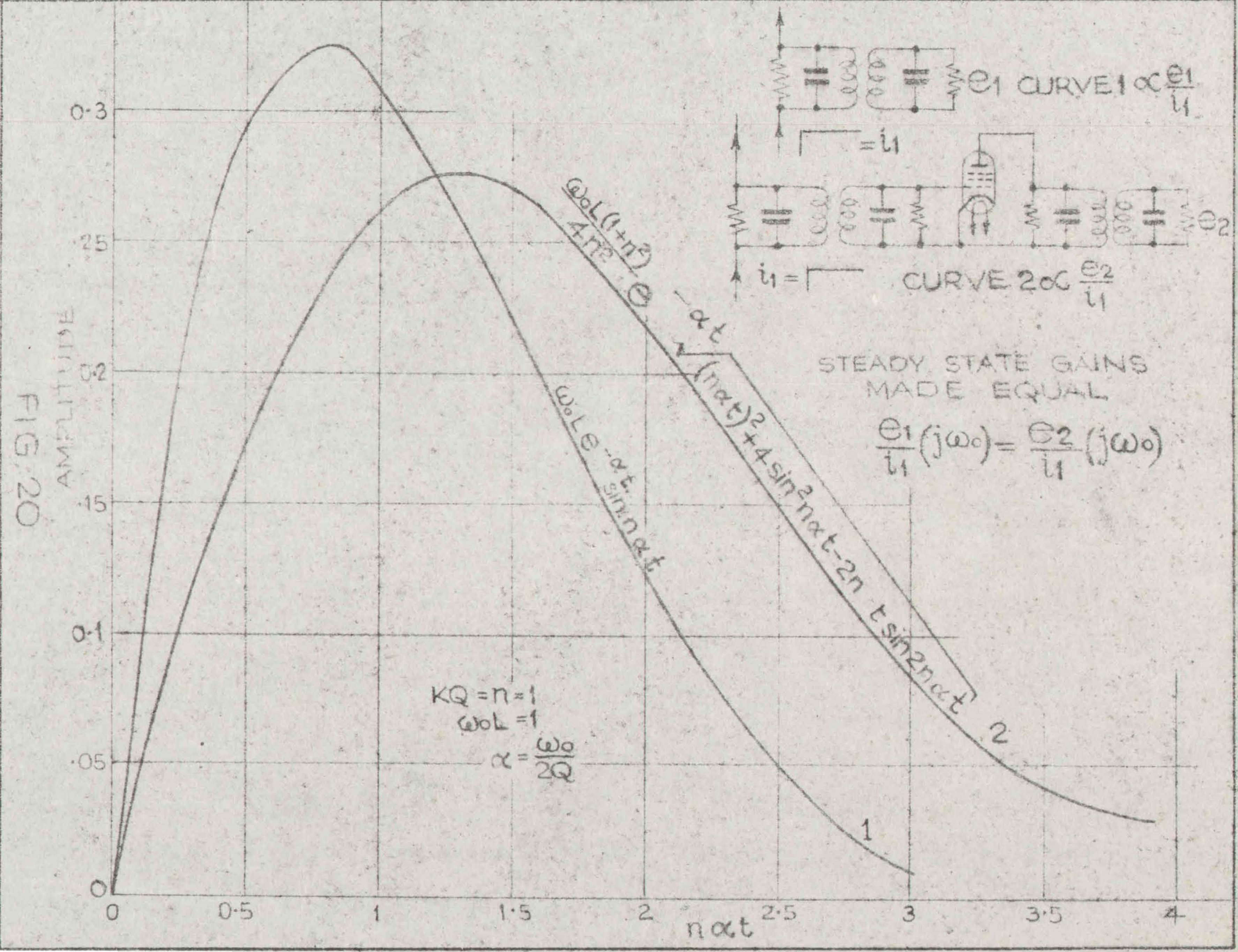
FIG. 20



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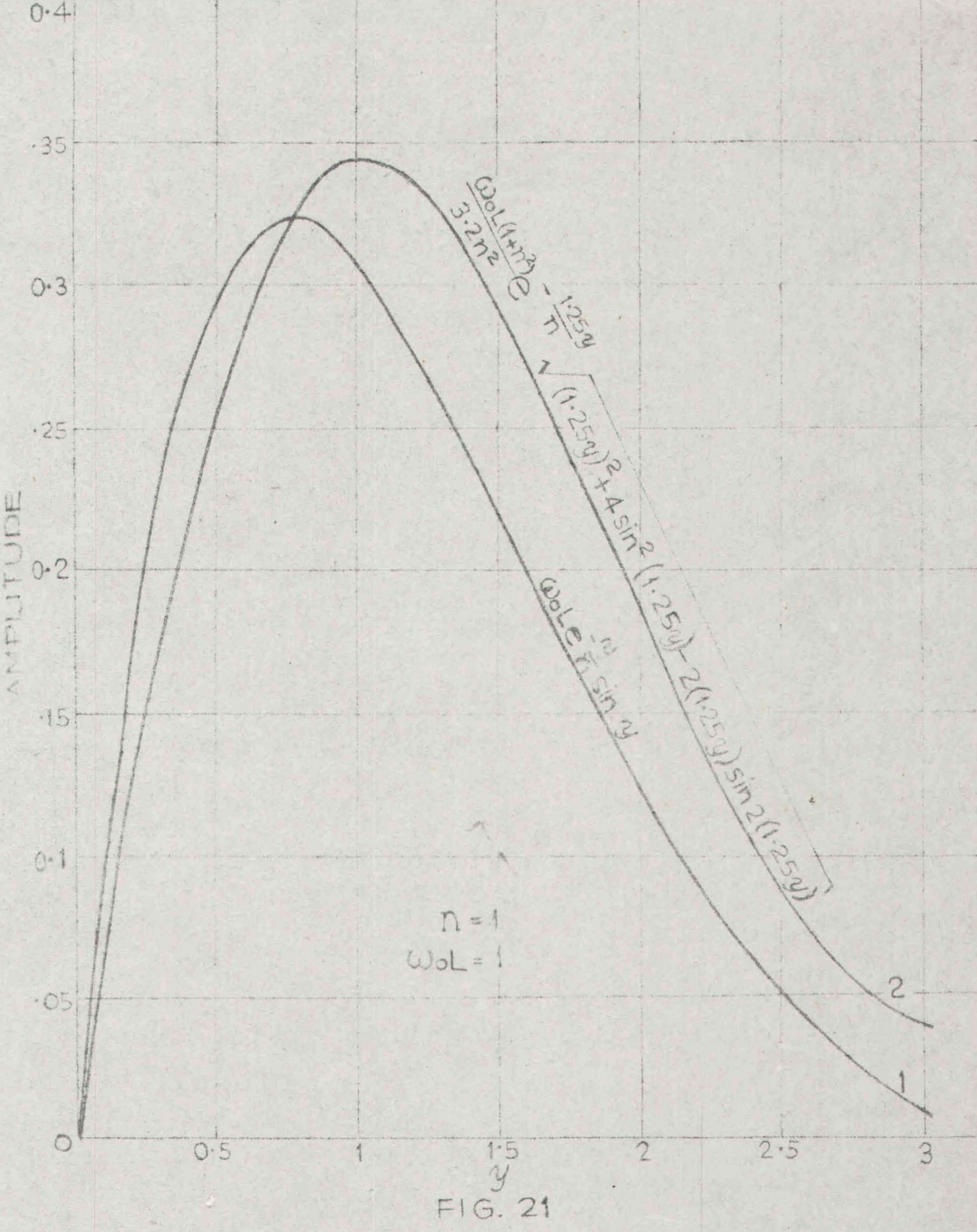

FIG. 21

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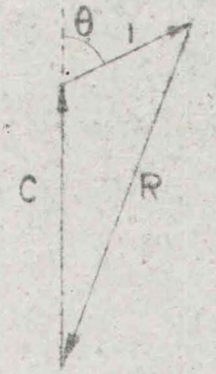

FIG.22

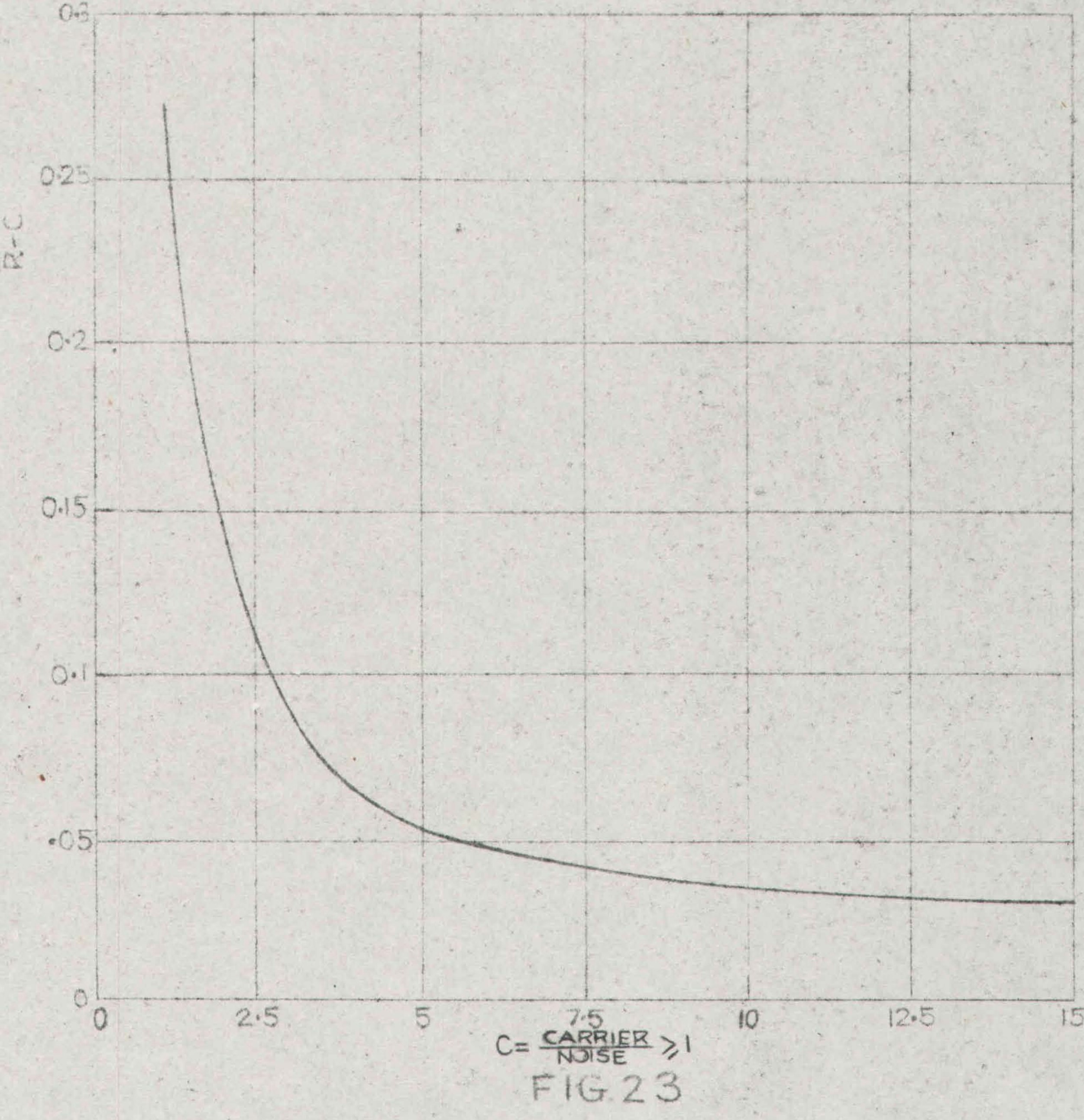

FIG.23

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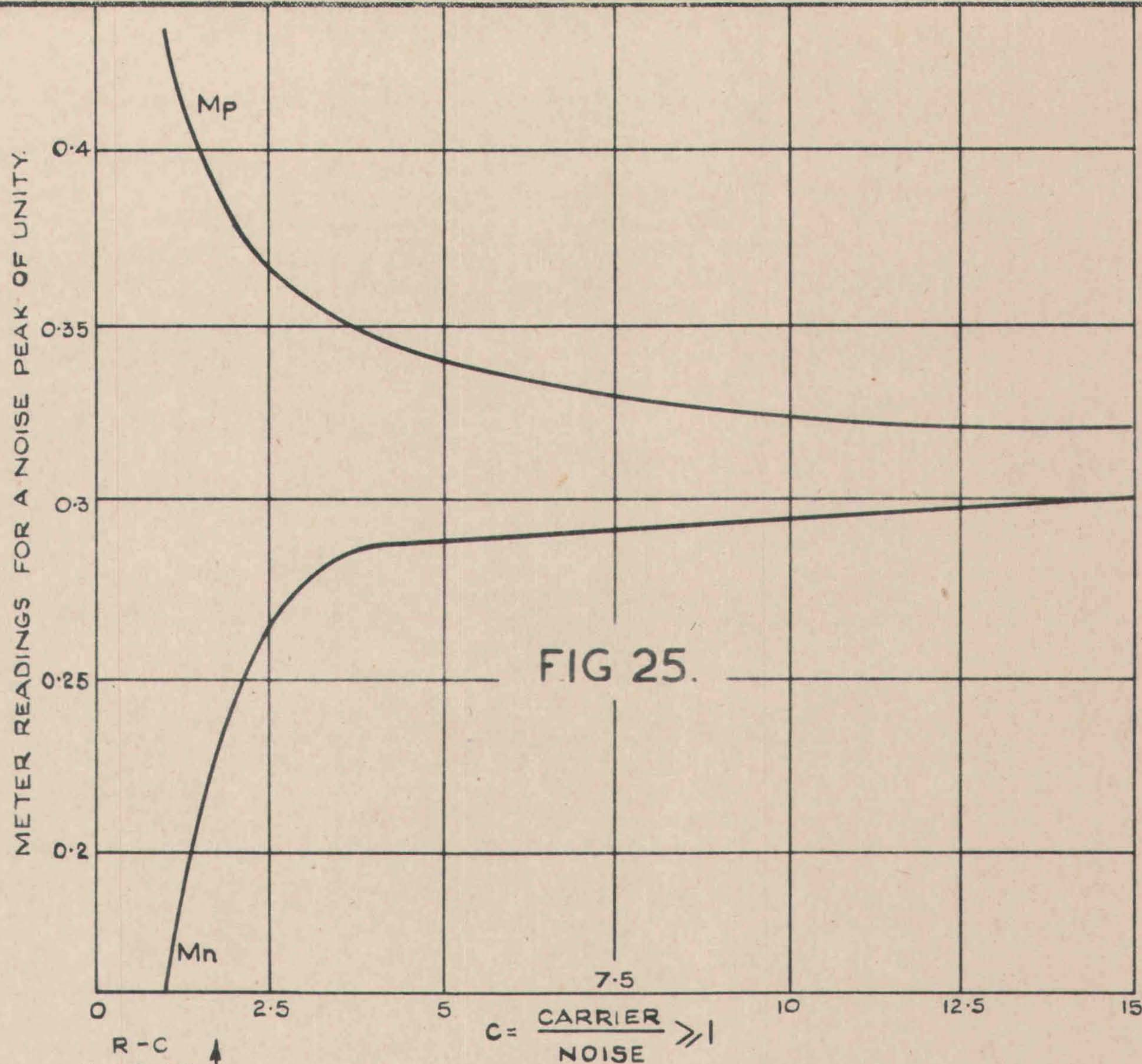

FIG 25.

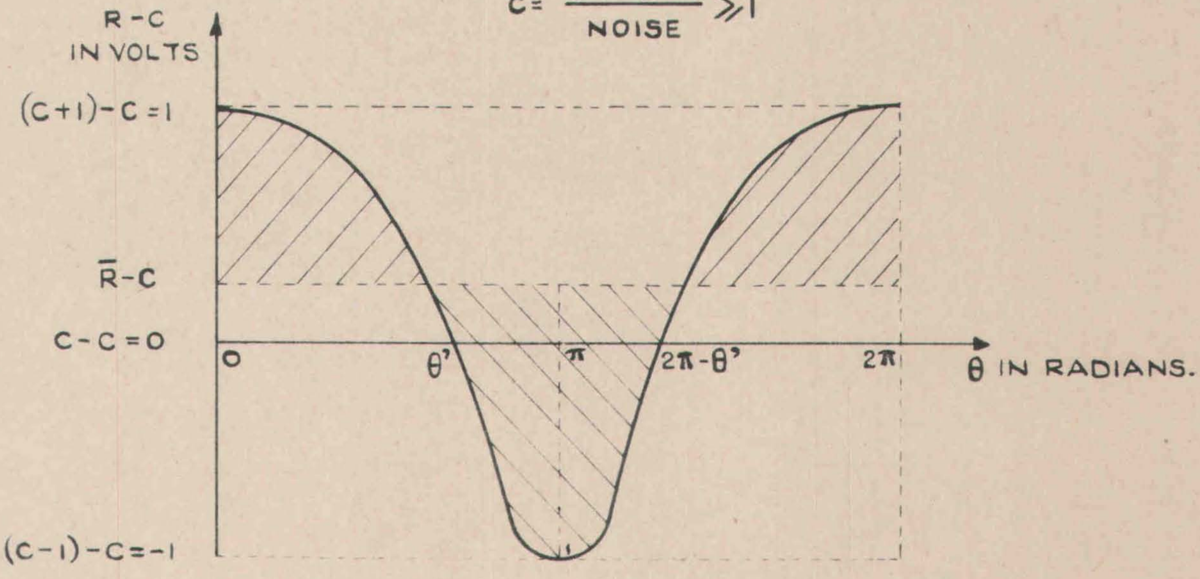

FIG. 24.

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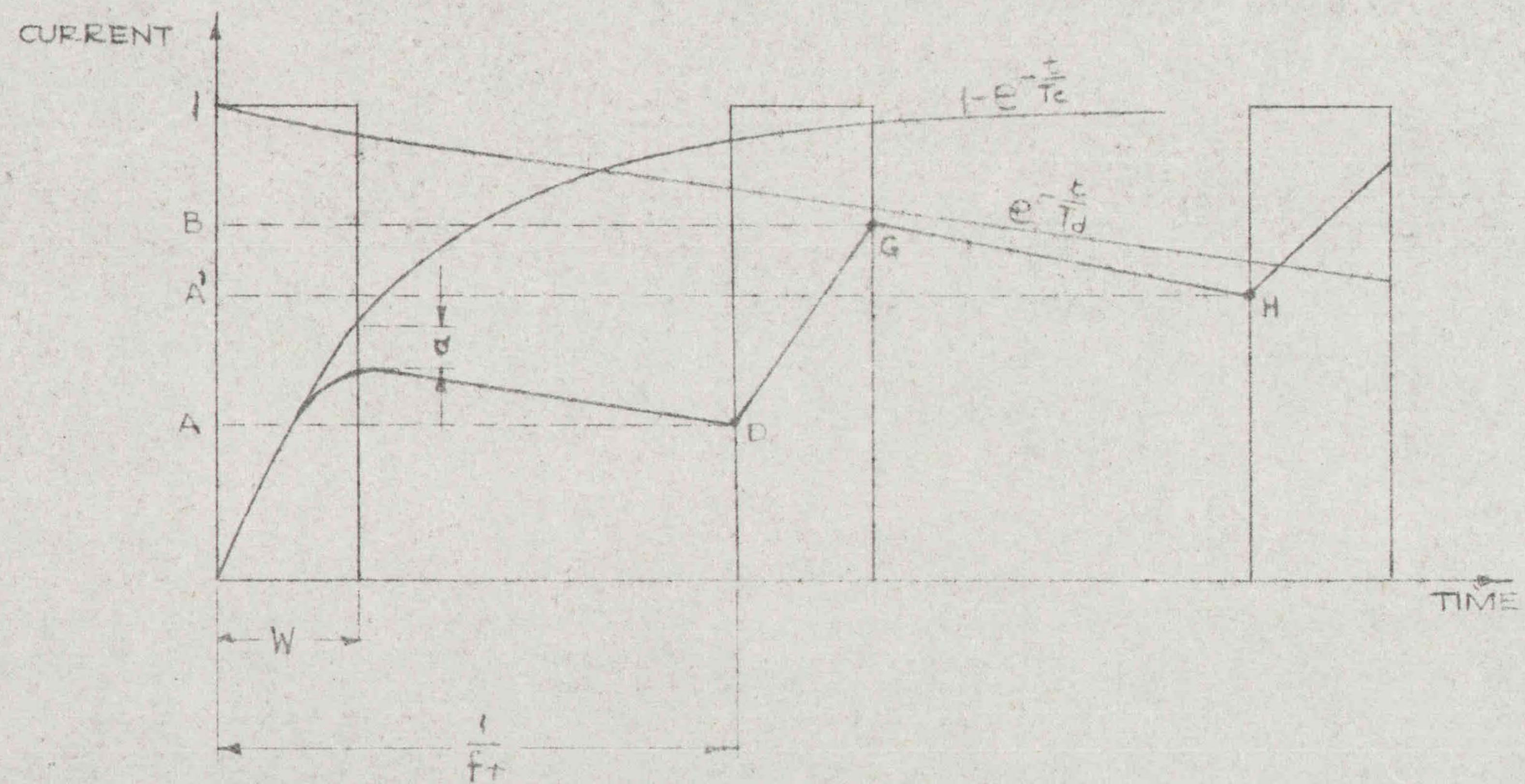

FIG. 26

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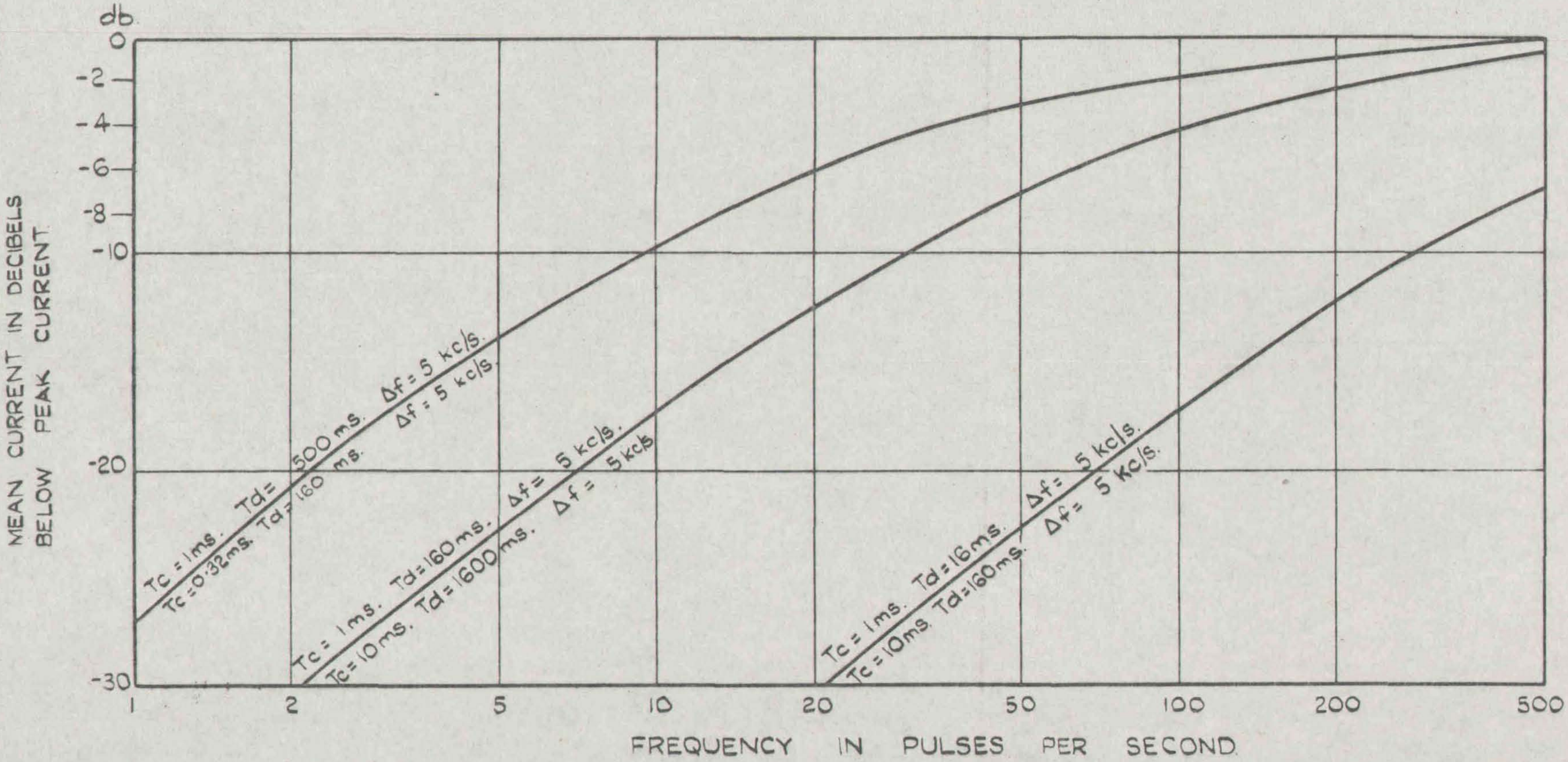

FIG. 27

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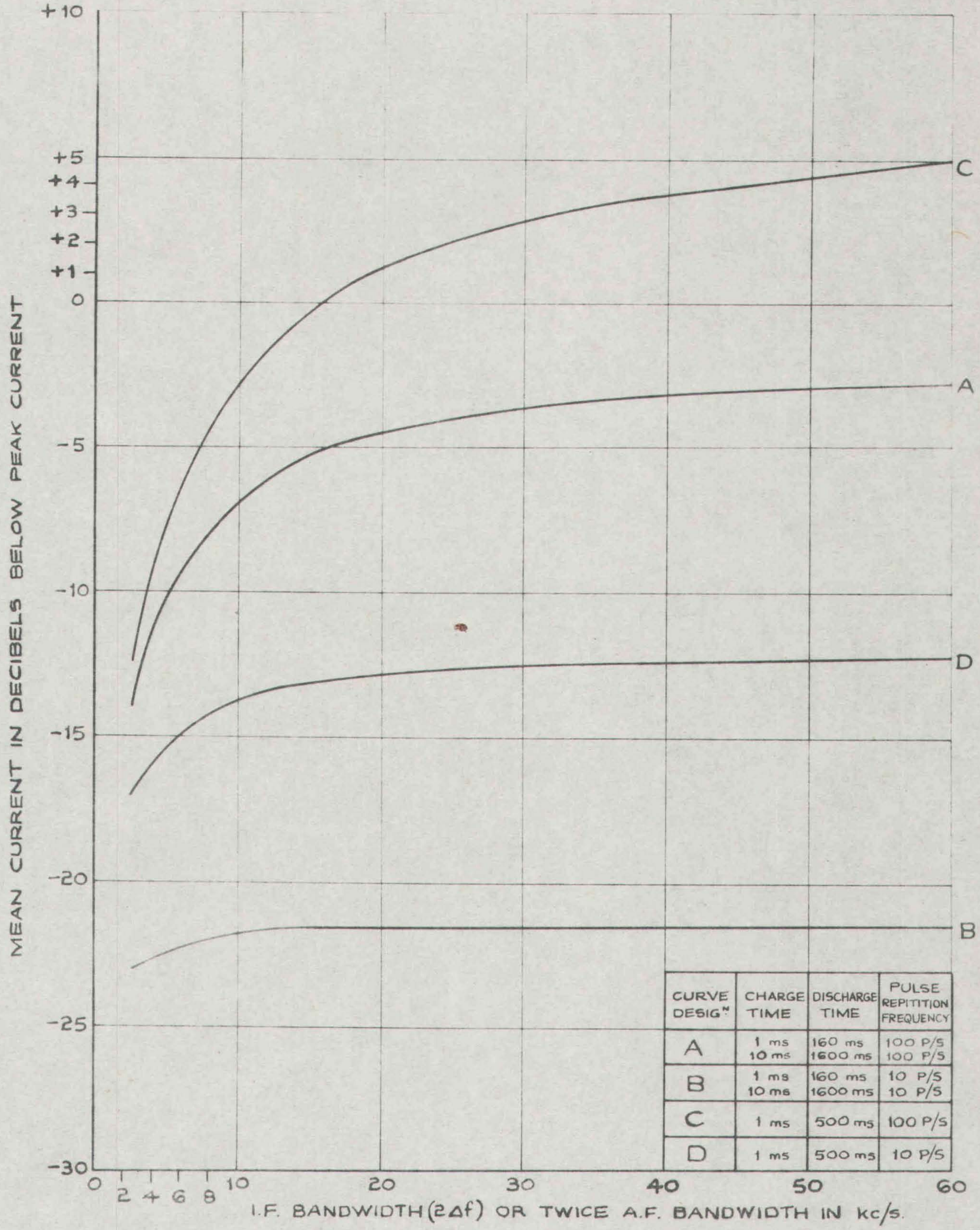

CURVE DESIG^N	CHARGE TIME	DISCHARGE TIME	PULSE REPITITION FREQUENCY
A	1 ms 10 ms	160 ms 1600 ms	100 P/S 100 P/S
B	1 ms 10 ms	160 ms 1600 ms	10 P/S 10 P/S
C	1 ms	500 ms	100 P/S
D	1 ms	500 ms	10 P/S

FIG. 28.

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FIG 3C 1ST STEP GENERATOR & TIME BASE

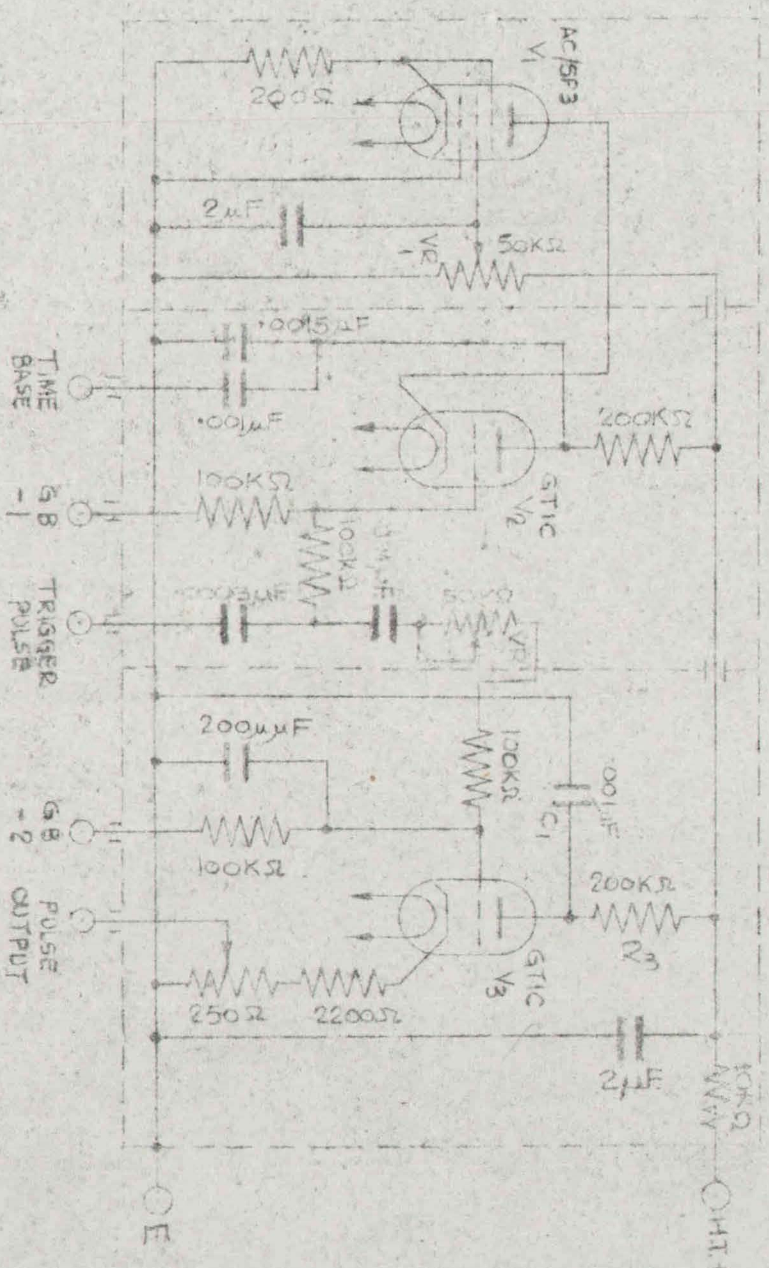
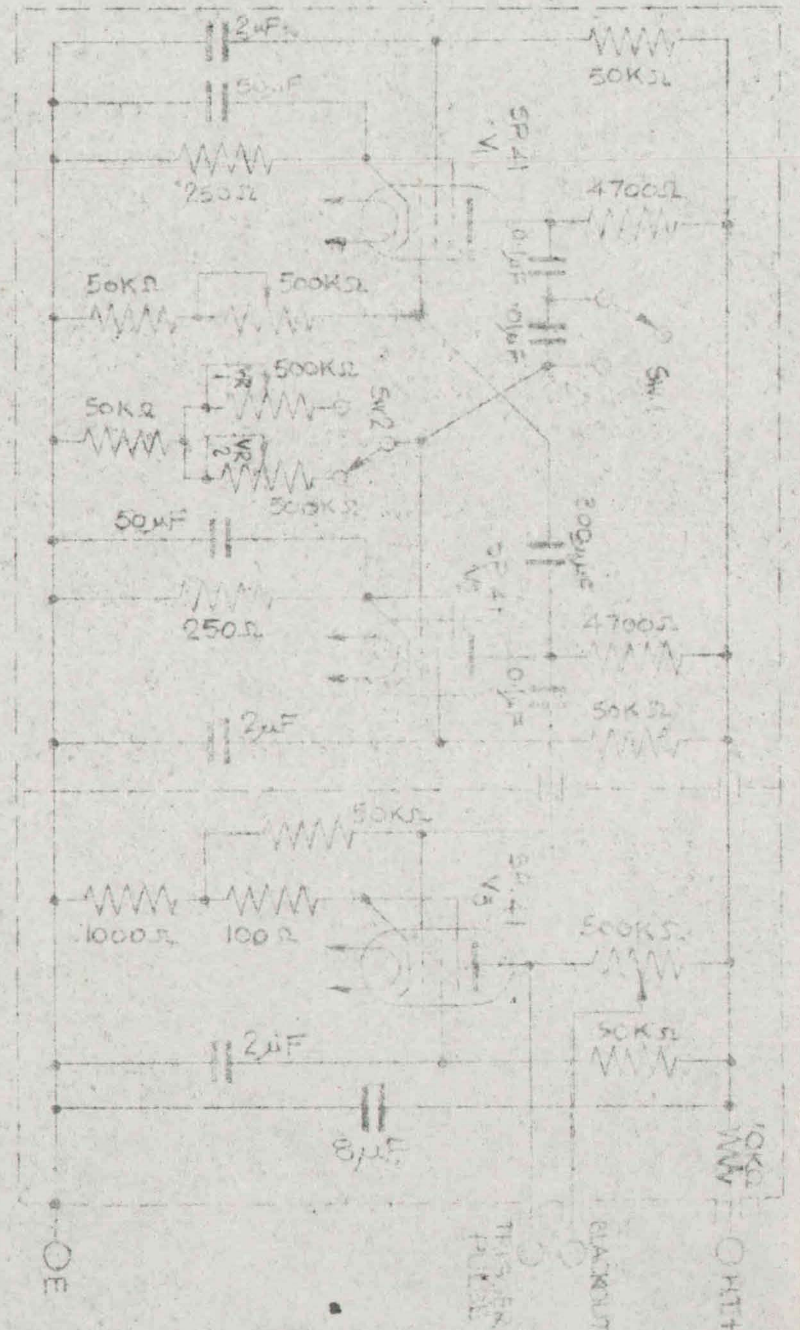


FIG 25 SQUARE WAVE GENERATOR



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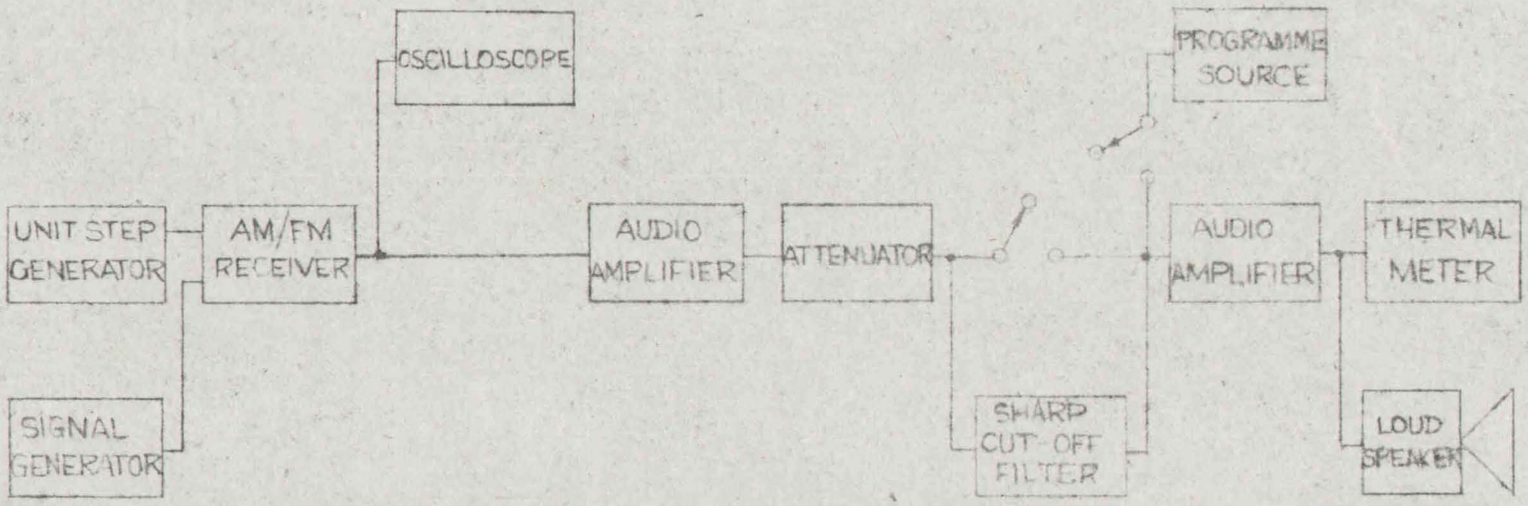

FIG. 31.

BLOCK SCHEMATIC OF EXPERIMENTAL LAYOUT

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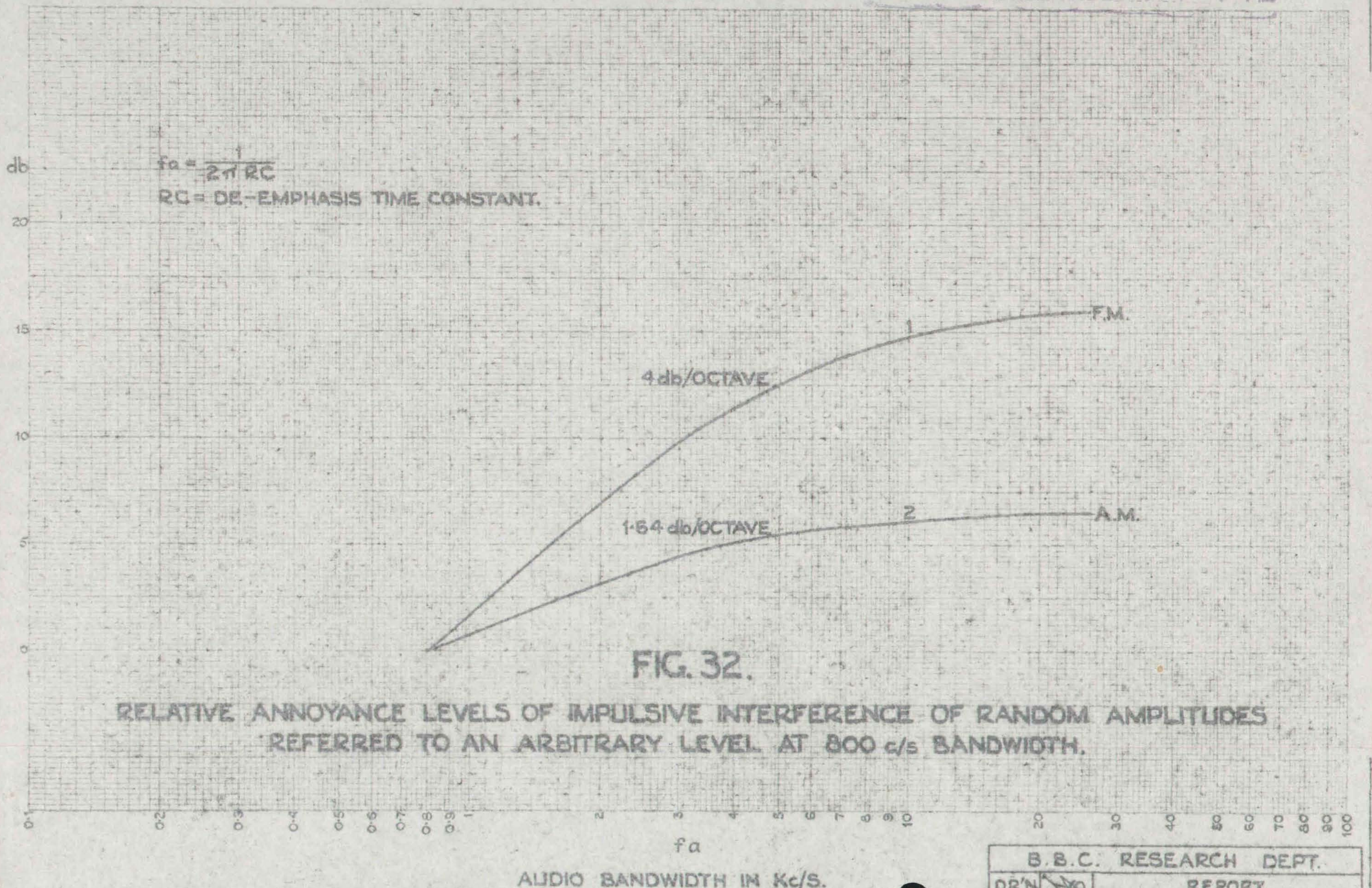

FIG. 32.

RELATIVE ANNOYANCE LEVELS OF IMPULSIVE INTERFERENCE OF RANDOM AMPLITUDES REFERRED TO AN ARBITRARY LEVEL AT 800 c/s BANDWIDTH.

IMPULSIVE INTERFERENCE IN A.M. AND F.M.

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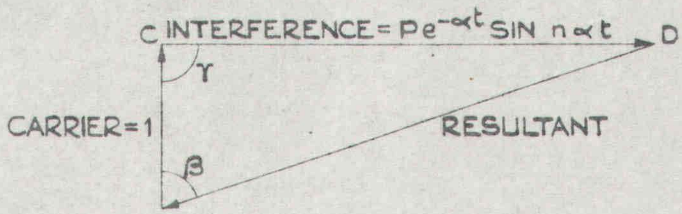

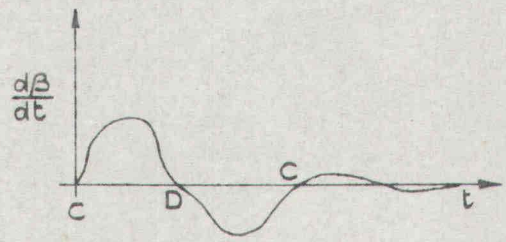

FIG. 33.

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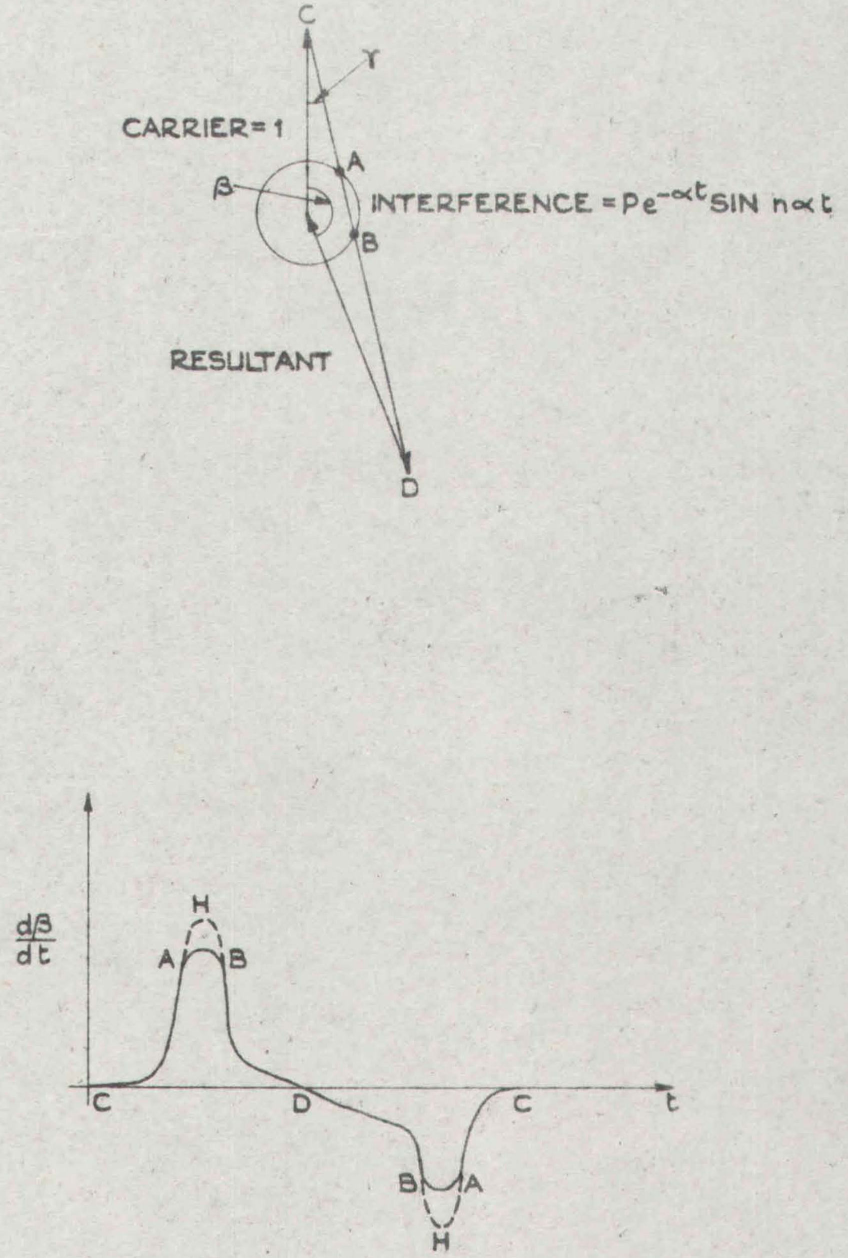

FIG. 34.

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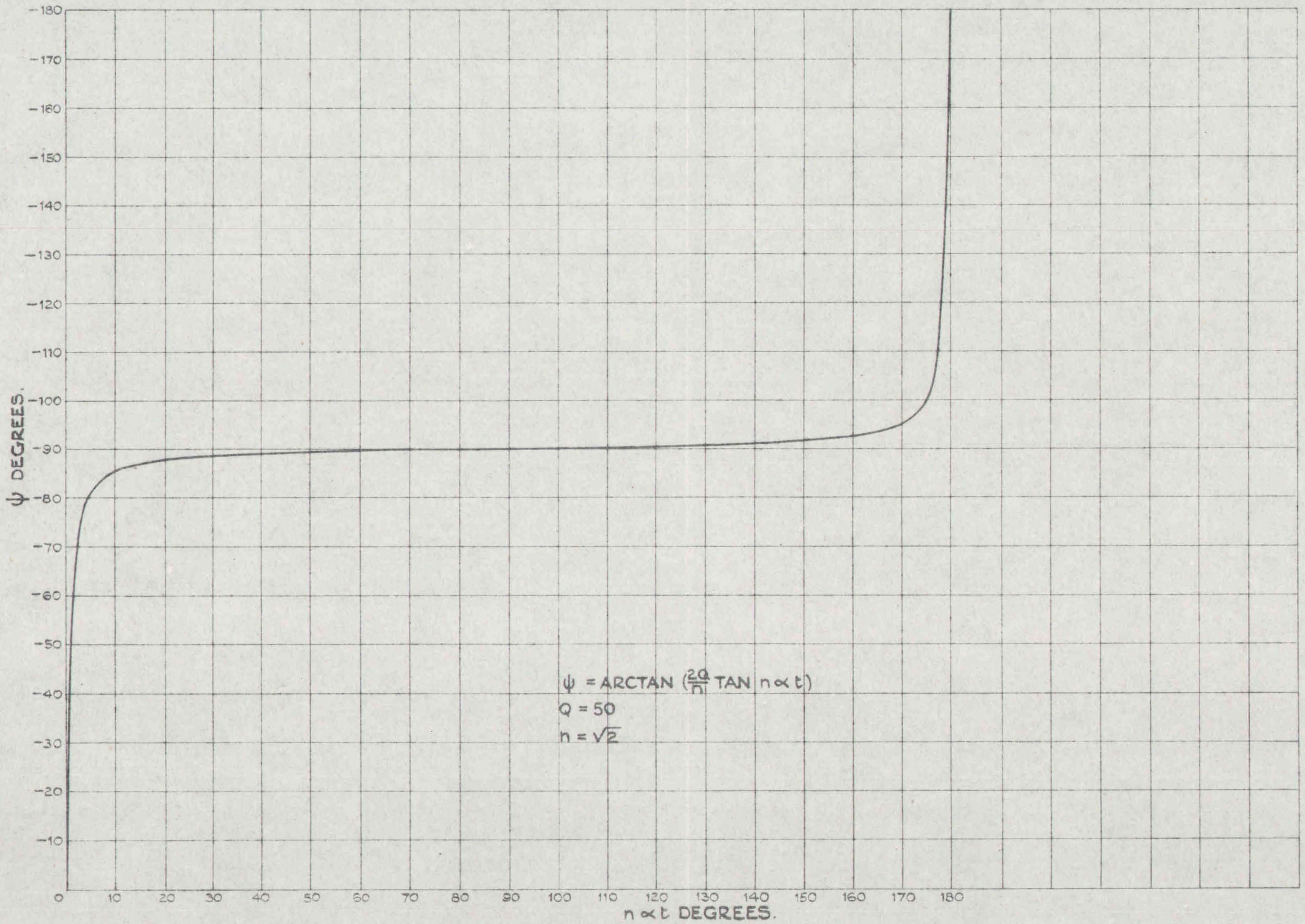

FIG. 35.

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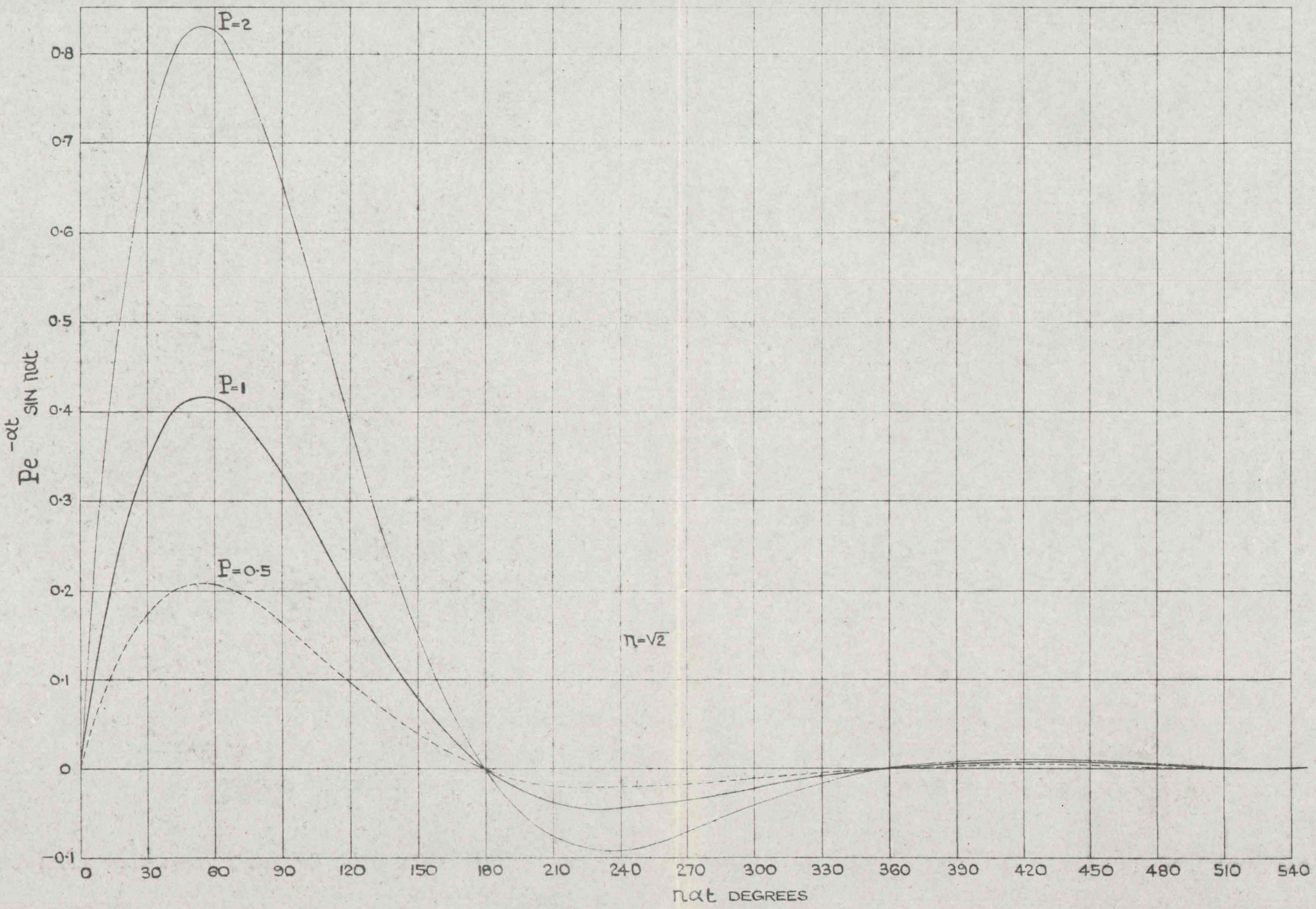

FIG. 36

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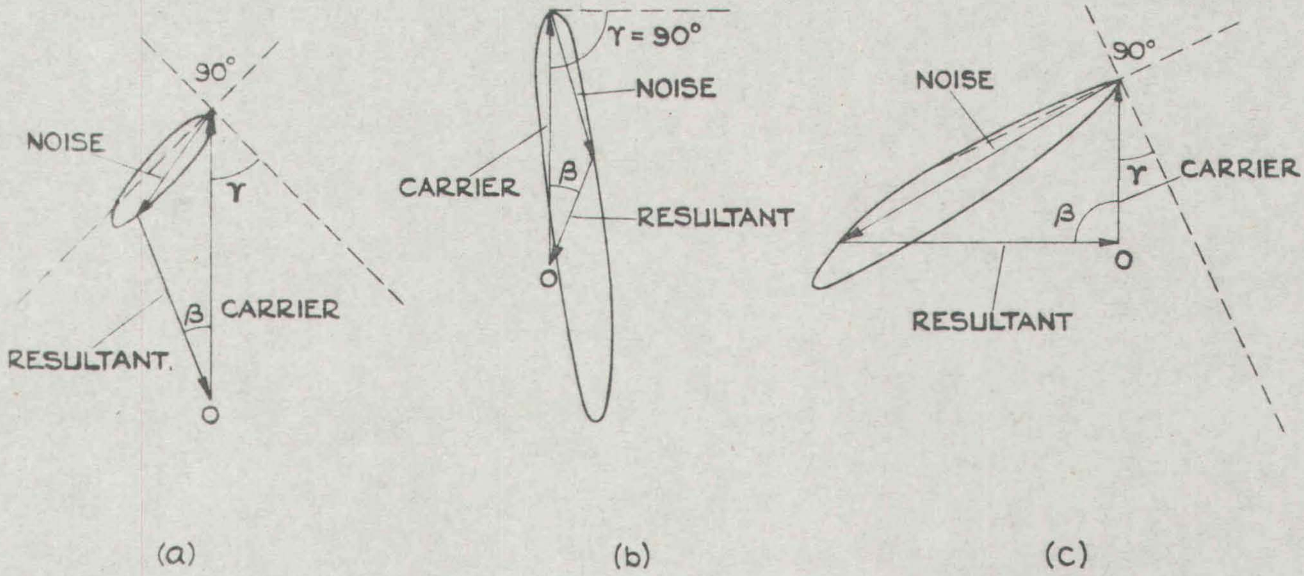

FIG. 37.

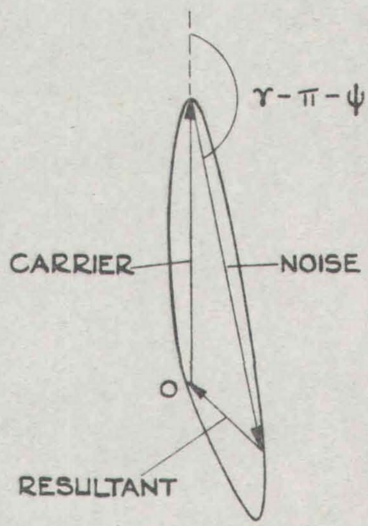

FIG. 41.

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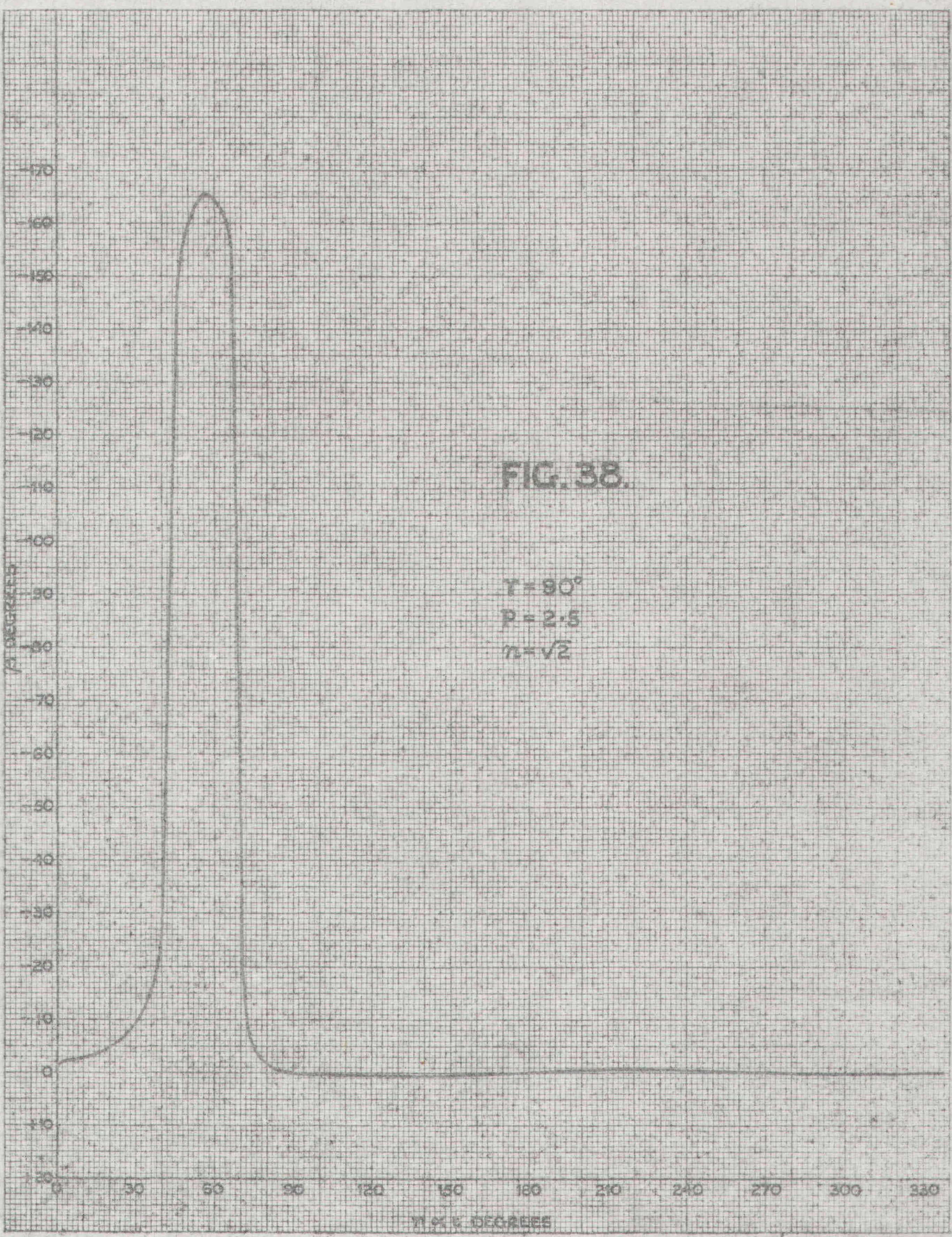

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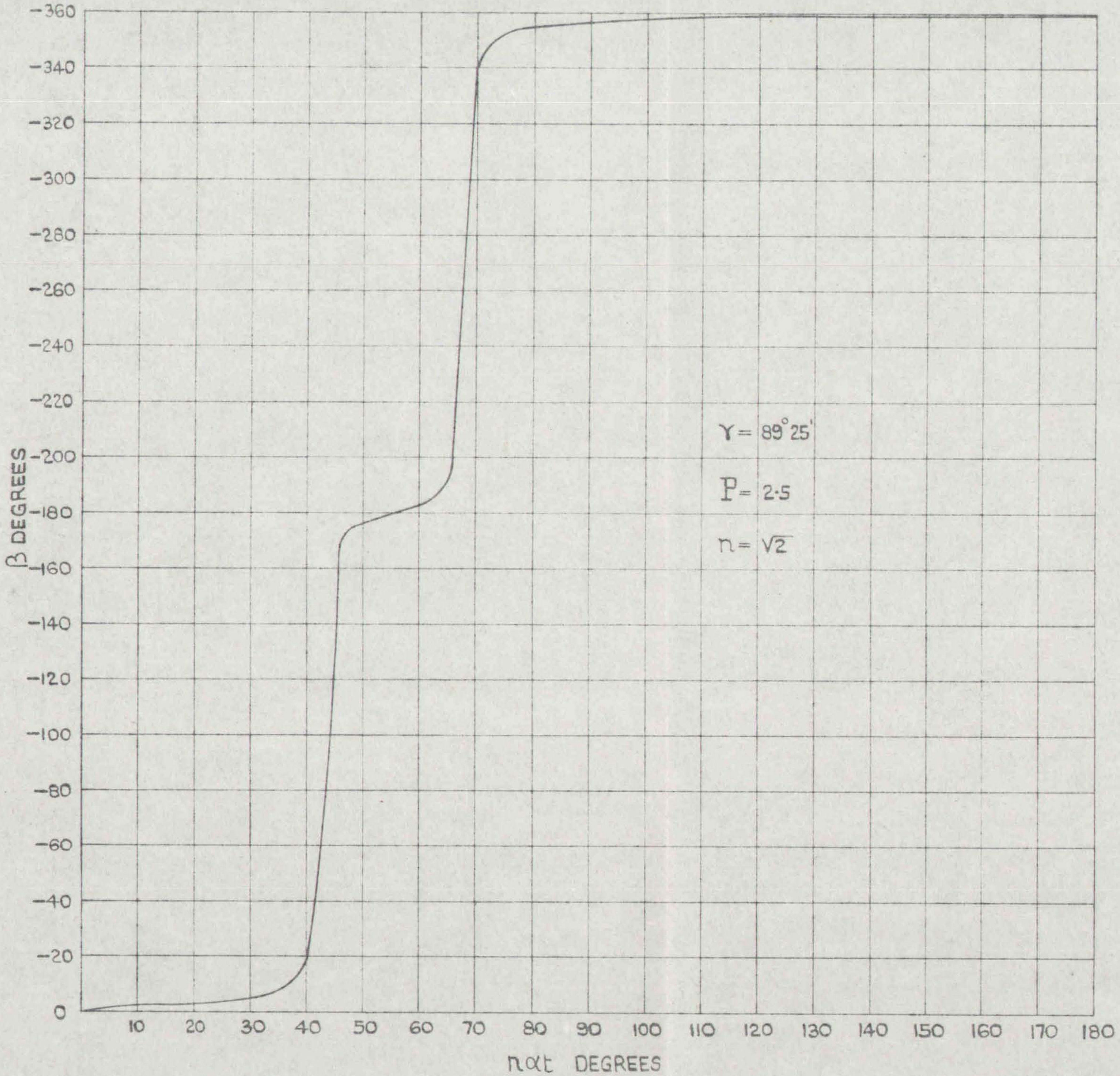

FIG.39

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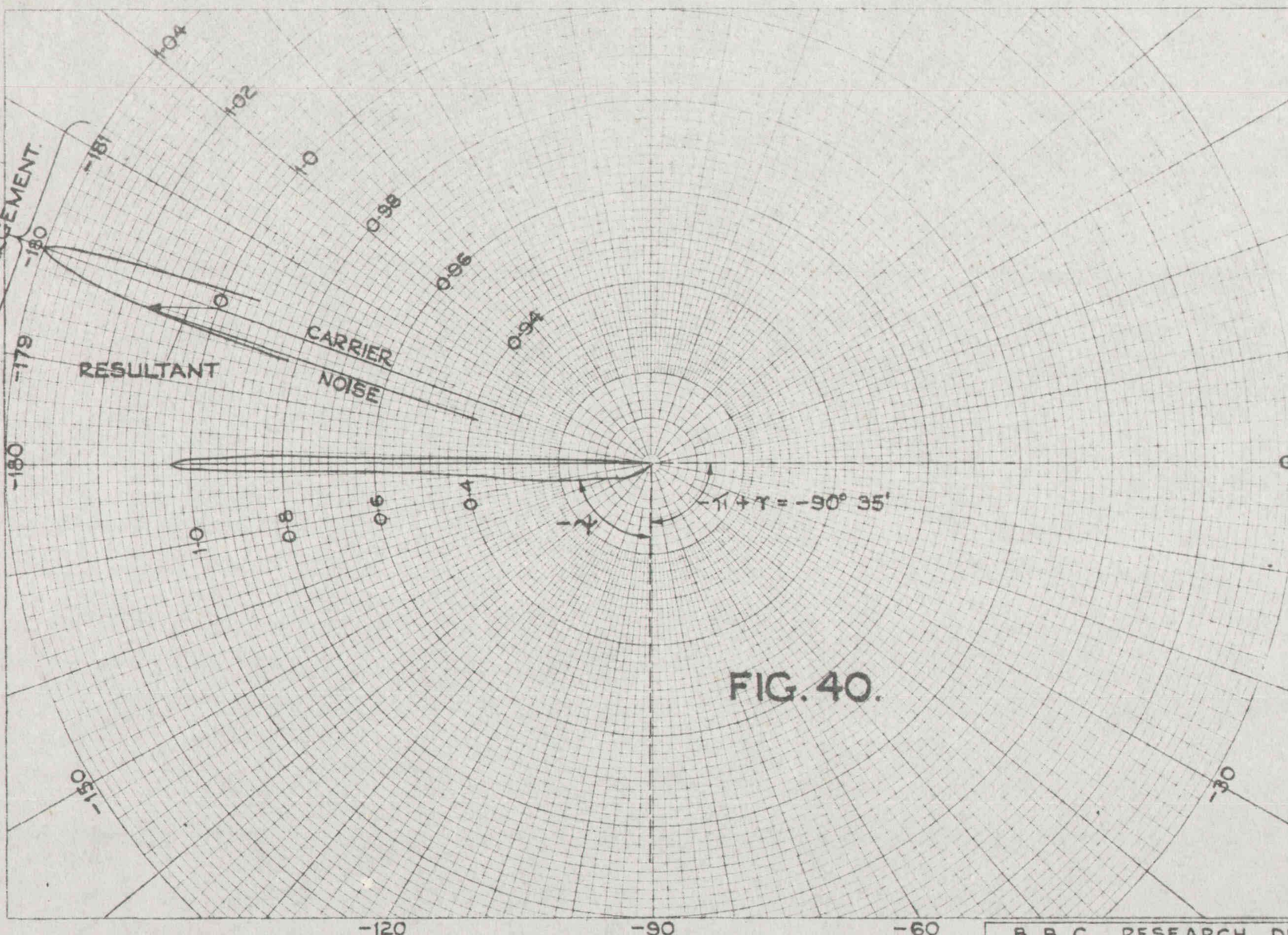

FIG. 40.

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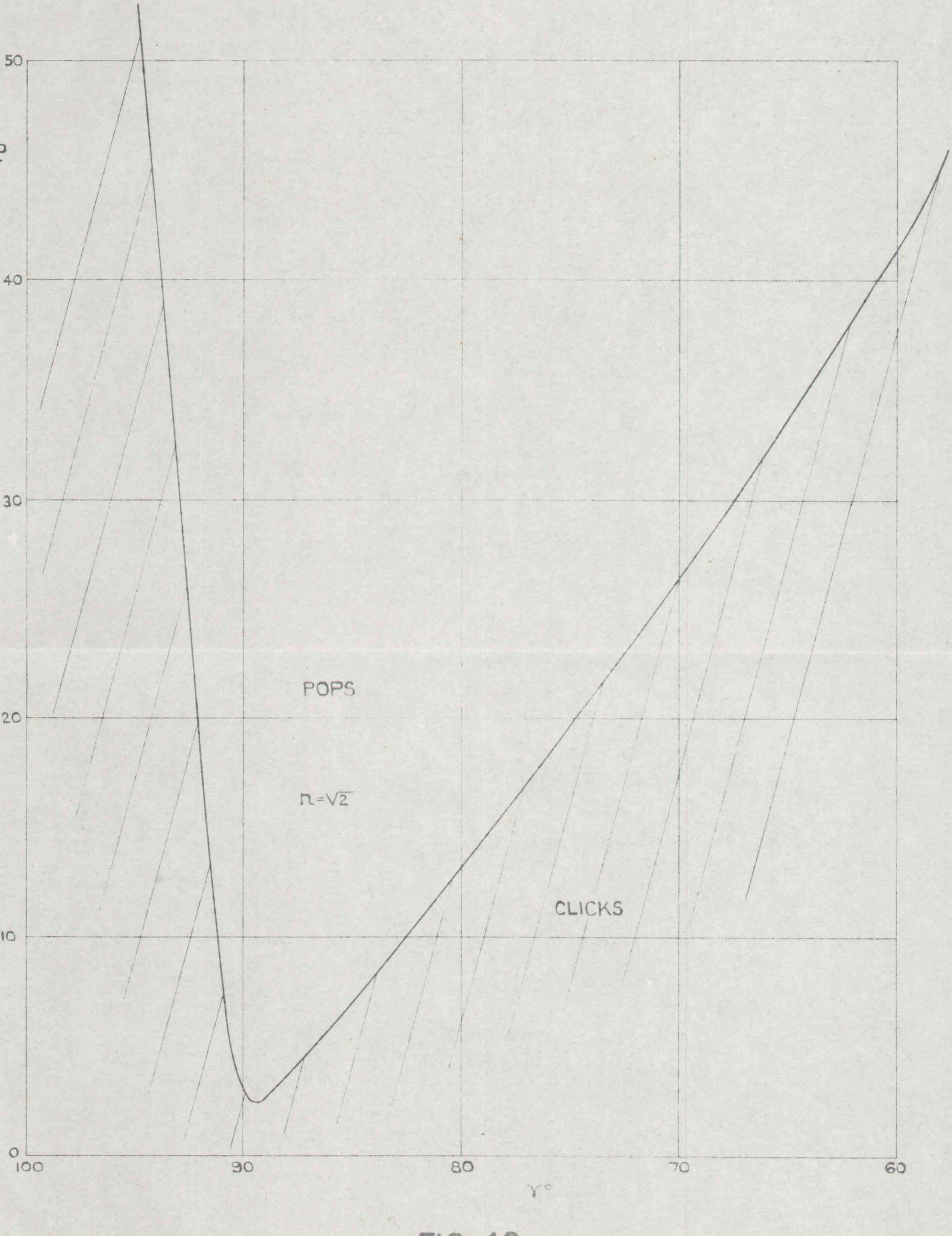

FIG. 42

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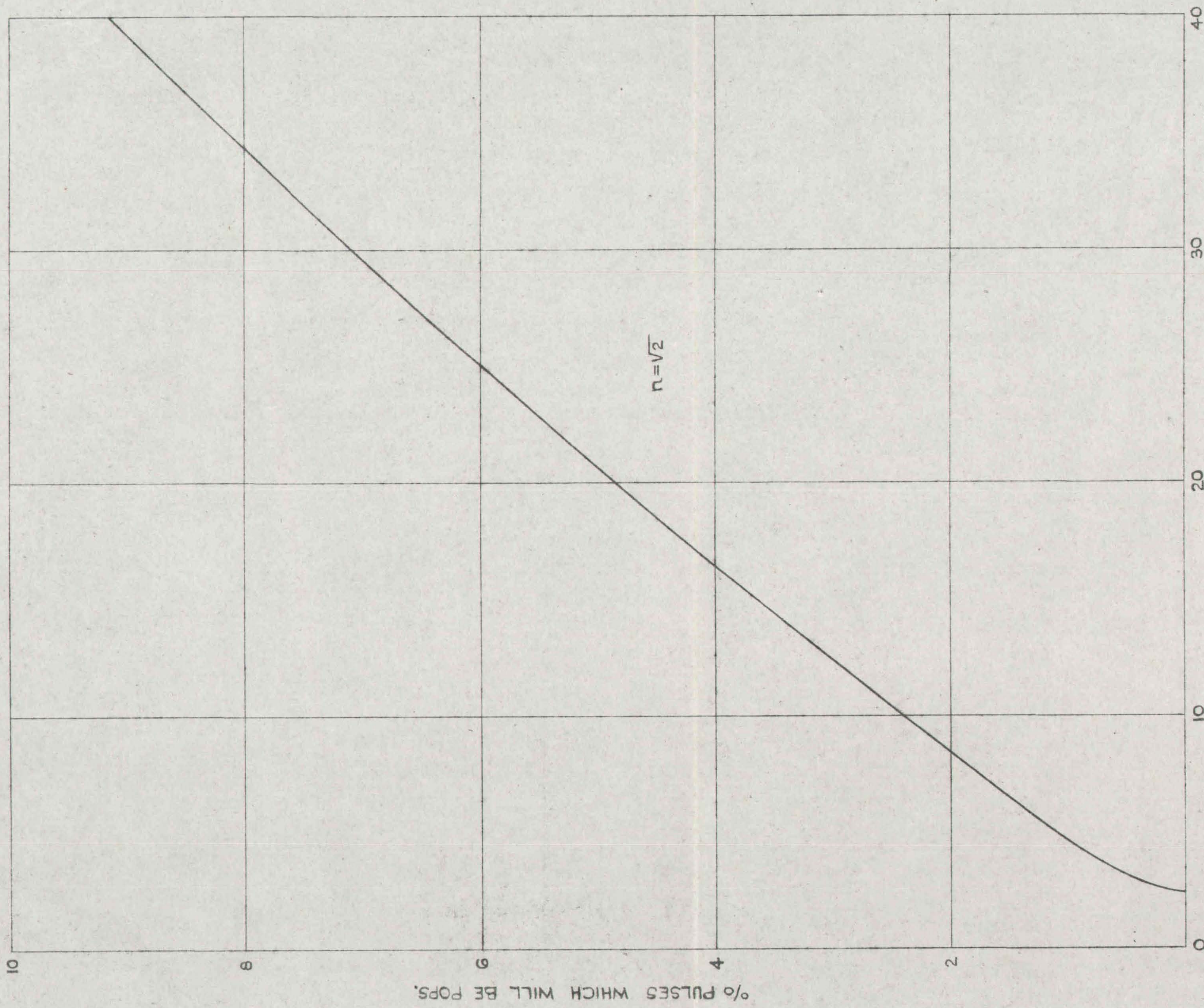

FIG. 43.

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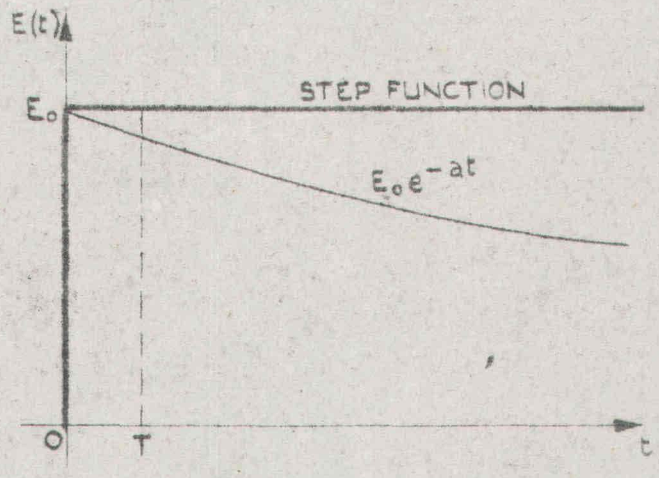

FIG. 44.

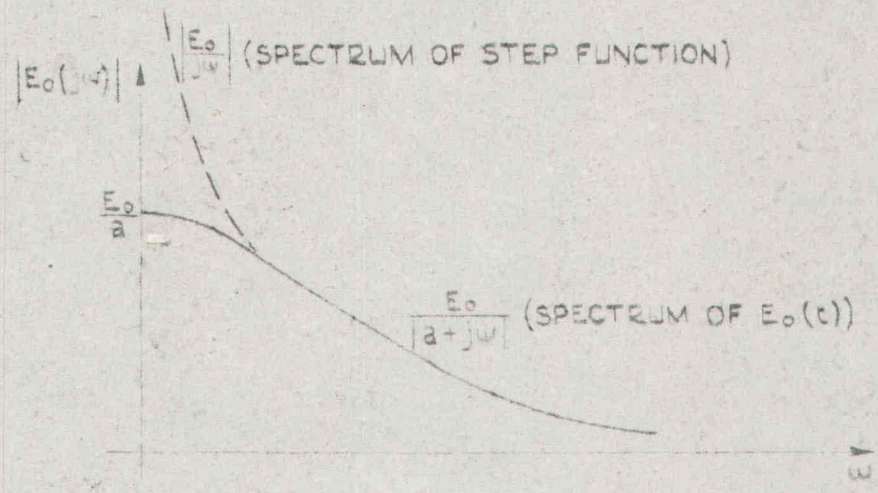

FIG. 45

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